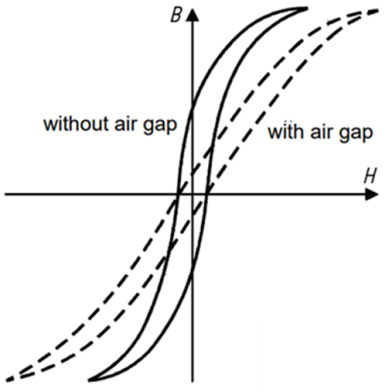
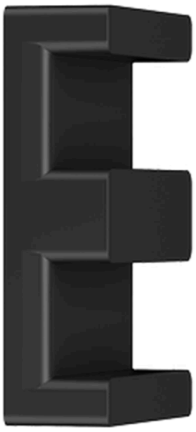
Gain, Lr and Lm characterization





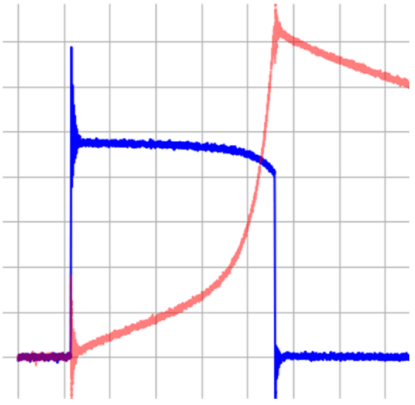


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  - Lr
  - Lm
  - Why is it difficult to fit the LRC curve?
- Small-signal characterization
  - Setup
- Operational gain measurement
  - Setup
  - Results
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Single pulse setup

Setup

For this test, we will use the same MOSFETs, driver, and microcontroller as those used in the LLC DC-DC converter. This test will serve as the reference for inductance characterization.

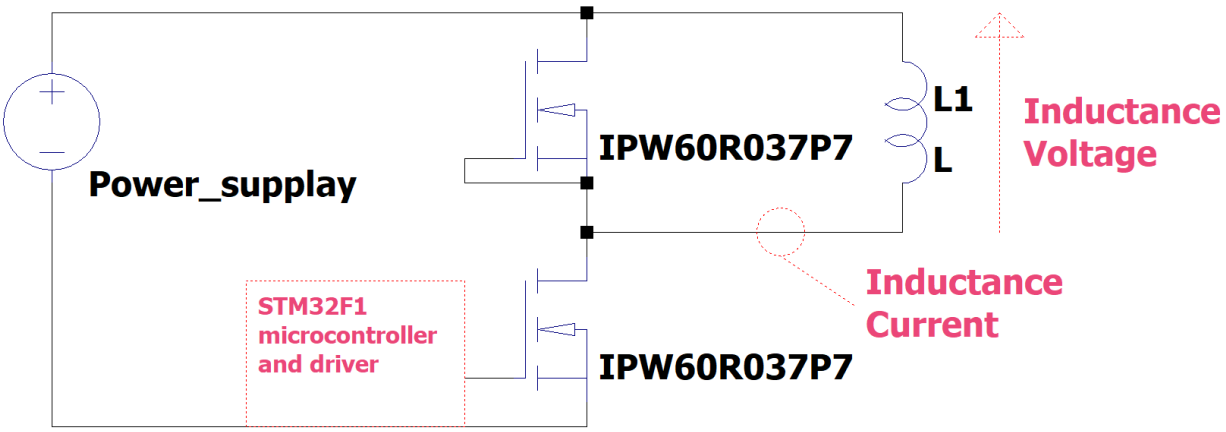


Figure 1: Single pulse setup (simplified schematic)

In the above simplified schematic, the DC link capacitor of 4 µF is not represented.
The Ton is controlled by the STM32F1 firmware.

Formulas used during preprocessing

- Magnetic Flux: $\phi = \frac{1}{N} \int V(t)\,\mathrm{d}t$
- Magnetic Flux Density: $B = \frac{\phi}{A_e}$
- Magnetic Field Strength: $H = \frac{N\,I_L}{l_e}$

Lm Without air gap

In this test, we will use the transformer choke with a PCB winding (4 turns), leaving the secondary open to characterize the Lm inductance. No air gap is used.
The main objective is to compare the measured inductance with the datasheet value of this choke.

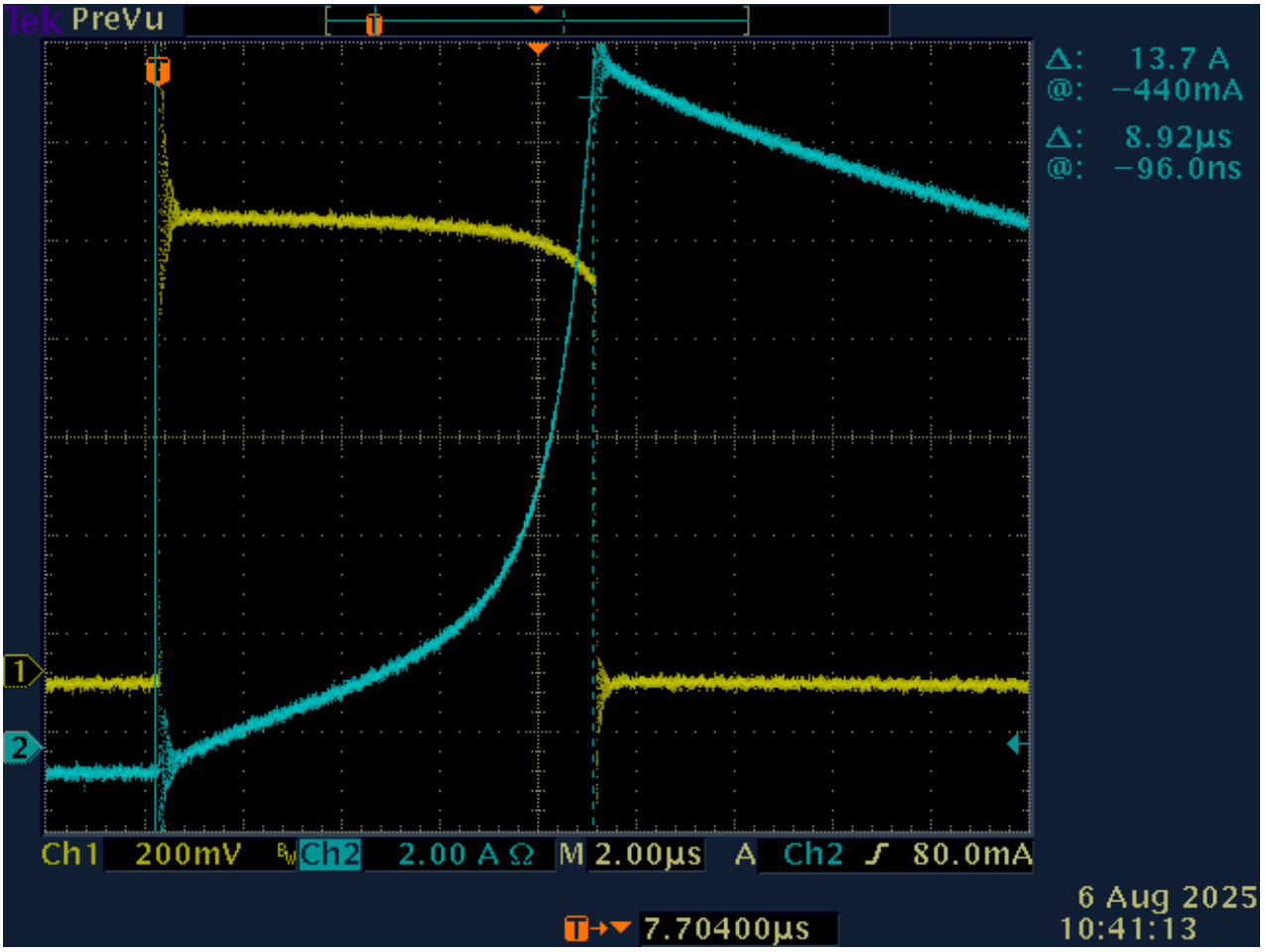
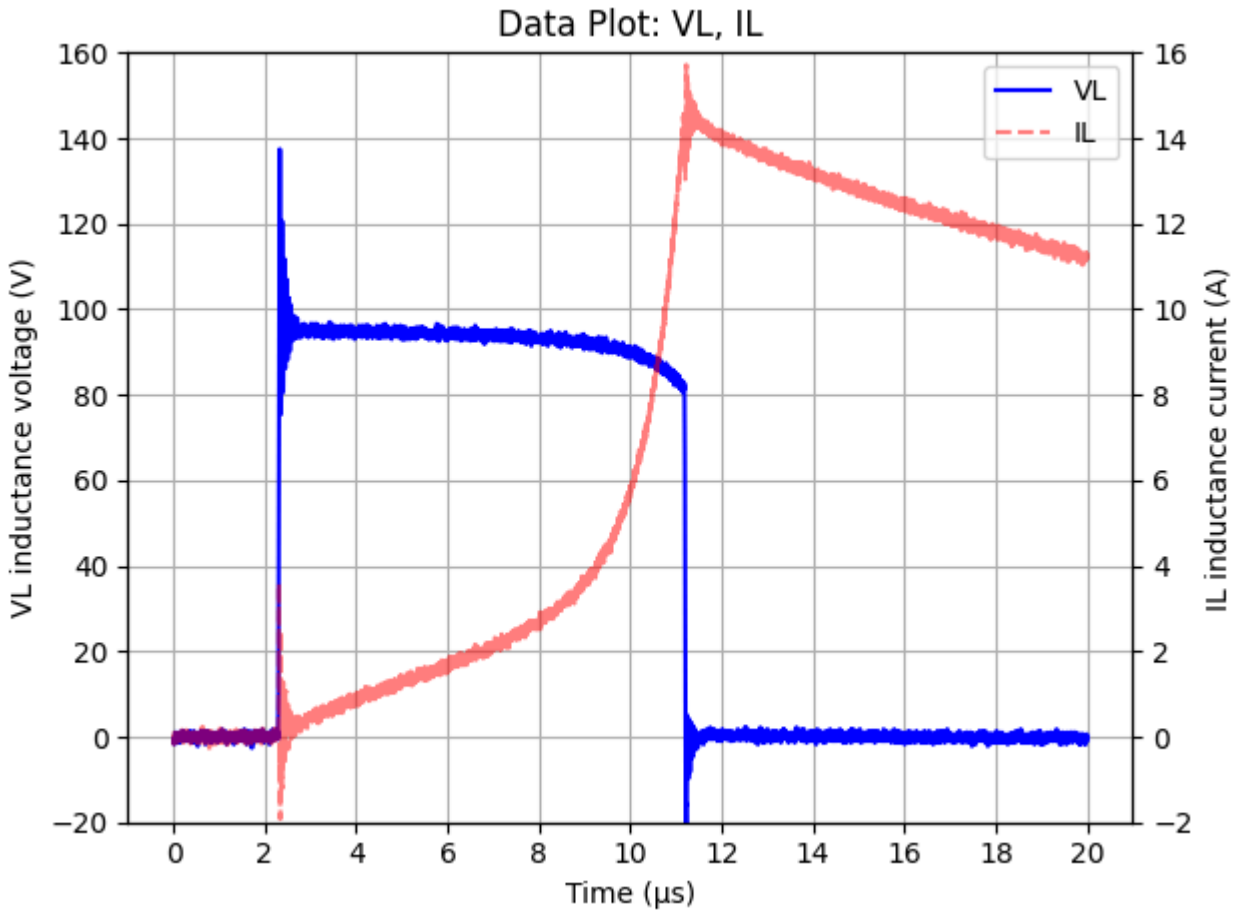


Figure 2: Oscilloscope screenshot, CH1 = V_L, CH2 = I_L

The plot shows the current and voltage data with zero correction at the beginning



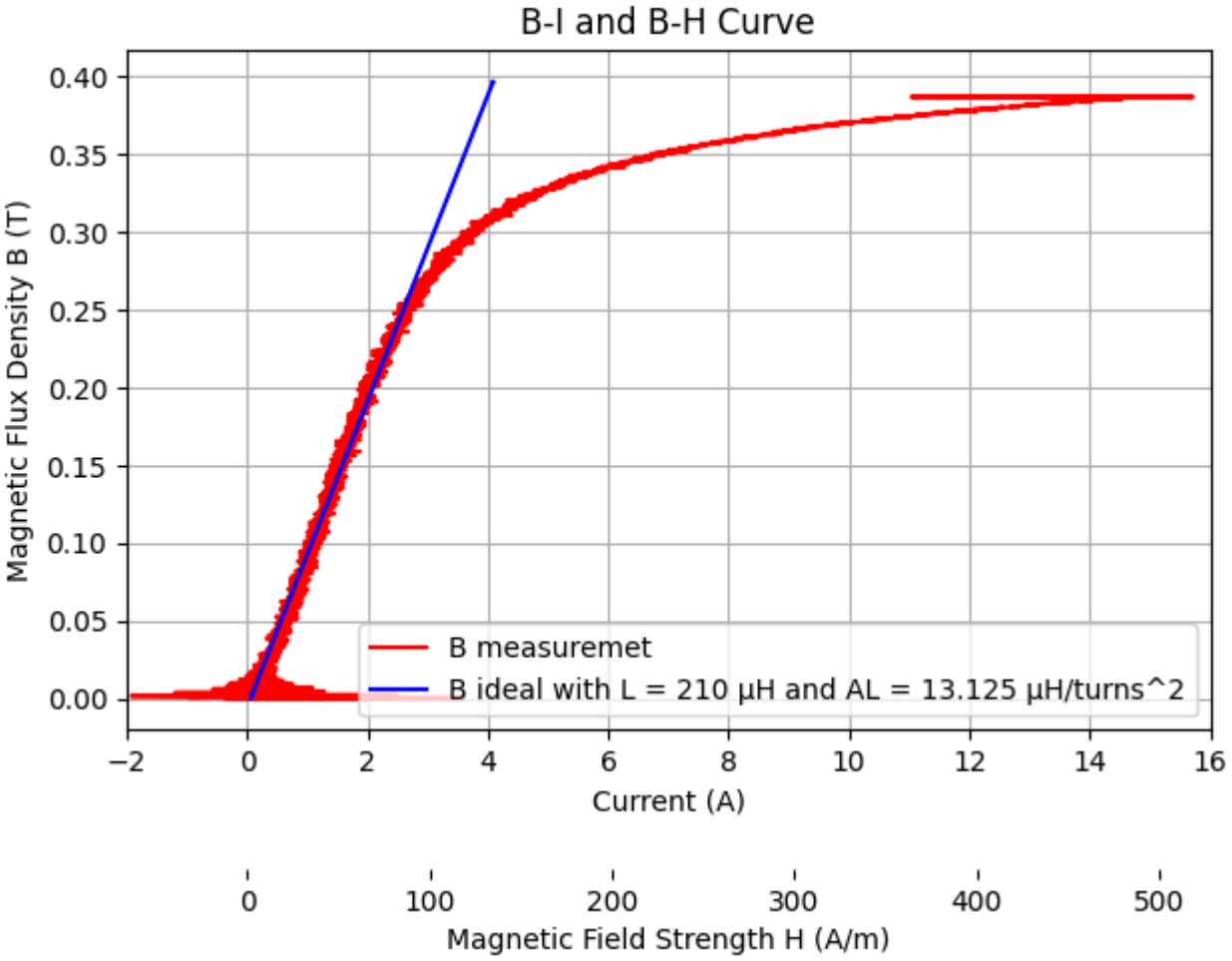
See fromulas above.

For more information, see the datasheet

$$A_e = 534.200 \ \ (\mathrm{mm2})$$

$$l_e = 121.200 \ \ (\mathrm{mm})$$

$$\mathrm{AL}_{nom} = 9600 \ \ (\mathrm{nH/N2})$$



Comparison between the measured value and the typical value provided in the datasheet:

$$\mathrm{Al}_{Measur} = 13125 \ \ (\mathrm{nH/turns2} : \text{see the above estimation})$$

$$\mathrm{Al}_{nominal} = 9600 \ \ (\mathrm{nH/turns2} : \text{see the datasheet})$$

$$\mathrm{Error}_{AL} = 100 \cdot \frac{\mathrm{Al}_{Measur} - \mathrm{Al}_{nominal}}{\mathrm{Al}_{nominal}} = 100 \cdot \frac{13125 - 9600}{9600} \qquad = 36.719 \ \ (\backslash\%)$$

The measured error (36.7%) exceeds the datasheet specification of 25%.

Estimating the inductance for a 100 µm air gap (2×50 µm) using the nominal AL value from the datasheet.

$$\mathrm{Ae}_{mm2} = 534.200 \ \ (\text{see the datasheet})$$

$$\mathrm{lg}_{um} = 100 \ \ (\mu\text{m } 2\text{x}50\ \mu\text{m})$$

$$\mathrm{lg} = 1 \times 10^{-6} \cdot \mathrm{lg}_{um} = 1 \times 10^{-6} \cdot 100 \qquad = 0.000 \ \ (0.10\text{e-}3 \ \ \text{Total gap } 2 * 0.05$$

$$\mathrm{Al}_{nom} = \mathrm{Al}_{nominal} \cdot 1 \times 10^{-9} = 9600 \cdot 1 \times 10^{-9} \qquad = 0$$

$$\begin{aligned}\mathrm{AL}_{gap_{nH1}} &= \frac{1 \times 10^{9}}{\left(\frac{1}{\mathrm{Al}_{nom}}\right) + \left(\frac{\mathrm{lg}}{\mu_0 \cdot \mathrm{Ae}_{mm2} \cdot 1 \times 10^{-6}}\right)} \\ &= \frac{1 \times 10^{9}}{\left(\frac{1}{0.000}\right) + \left(\frac{0.000}{0.000 \cdot 534.200 \cdot 1 \times 10^{-6}}\right)} \\ &= 3950.502 \ \ (\mathrm{nH} \ / \ \text{turns } 2)\end{aligned}$$

$$\mathrm{Lnew1} = 1 \times 10^{-3} \cdot (N)^2 \cdot \mathrm{AL}_{gap_{nH1}} = 1 \times 10^{-3} \cdot (4)^2 \cdot 3950.502 \qquad = 63.208$$

Estimating the inductance for a 100 µm air gap (2×50 µm) using the measured AL value

$$\begin{aligned}\mathrm{AL}_{gap_{nH2}} &= \frac{1 \times 10^{9}}{\left(\frac{1 \times 10^{9}}{\mathrm{Al}_{Measur}}\right) + \left(\frac{\mathrm{lg}}{\mu_0 \cdot \mathrm{Ae}_{mm2} \cdot 1 \times 10^{-6}}\right)} \\ &= \frac{1 \times 10^{9}}{\left(\frac{1 \times 10^{9}}{13125}\right) + \left(\frac{0.000}{0.000 \cdot 534.200 \cdot 1 \times 10^{-6}}\right)} \\ &= 4441.362 \ \ (\mathrm{nH} \ / \ \text{turns } 2)\end{aligned}$$

$$\mathrm{Lnew2} = 1 \times 10^{-3} \cdot (N)^2 \cdot \mathrm{AL}_{gap_{nH2}} = 1 \times 10^{-3} \cdot (4)^2 \cdot 4441.362 \qquad = 71.062 \ \ (\mu\mathrm{H})$$

Calculating the error between the datasheet air-gapped value and the measured AL value :

$$\mathrm{Error} = 100 \cdot \frac{\mathrm{AL}_{gap_{nH2}} - \mathrm{AL}_{gap_{nH1}}}{\mathrm{AL}_{gap_{nH1}}} = 100 \cdot \frac{4441.362 - 3950.502}{3950.502} \qquad = 12.425 \ \ (\backslash\%)$$

The dispersion error with the air gap is 12.5%, representing roughly one-third of the original 36.7% dispersion without an air gap.

Lm With Air gap

We will use 0.050 mm thick Kapton to form the air gap, resulting in a total air gap of 2×50 µm = 100 µm.



Figure 3: 0.05 mm Kapton layer as transformer air gap, total air gap 0.1mm

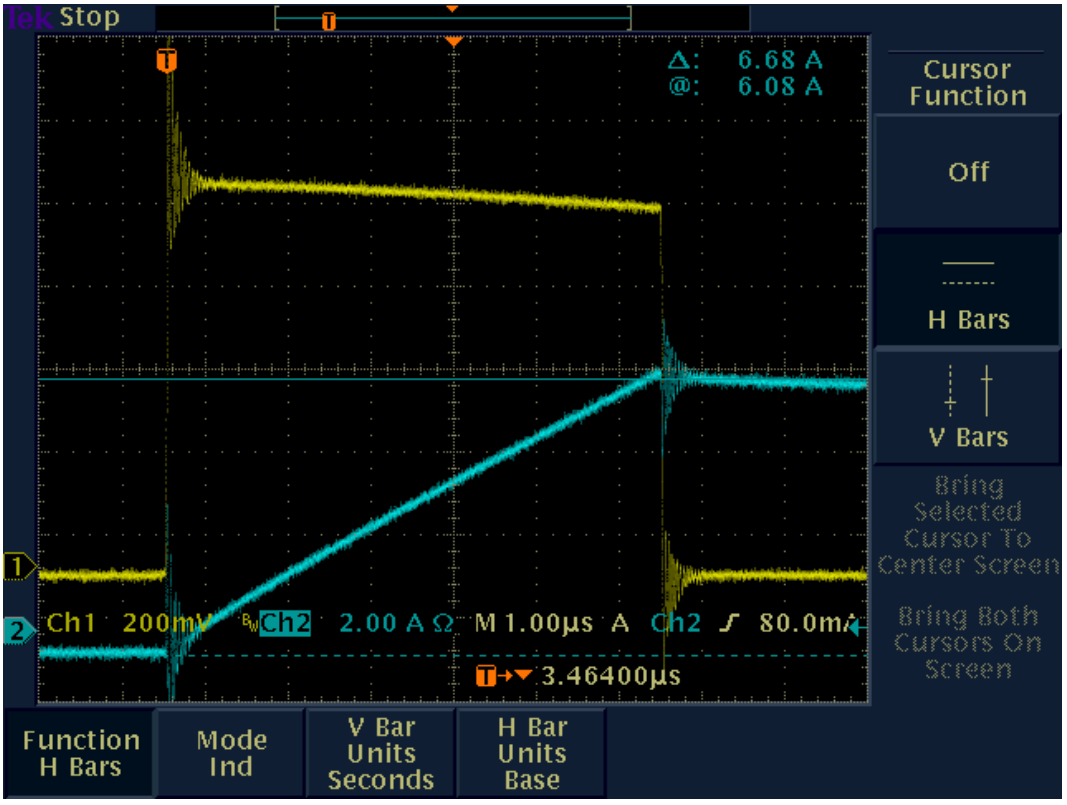
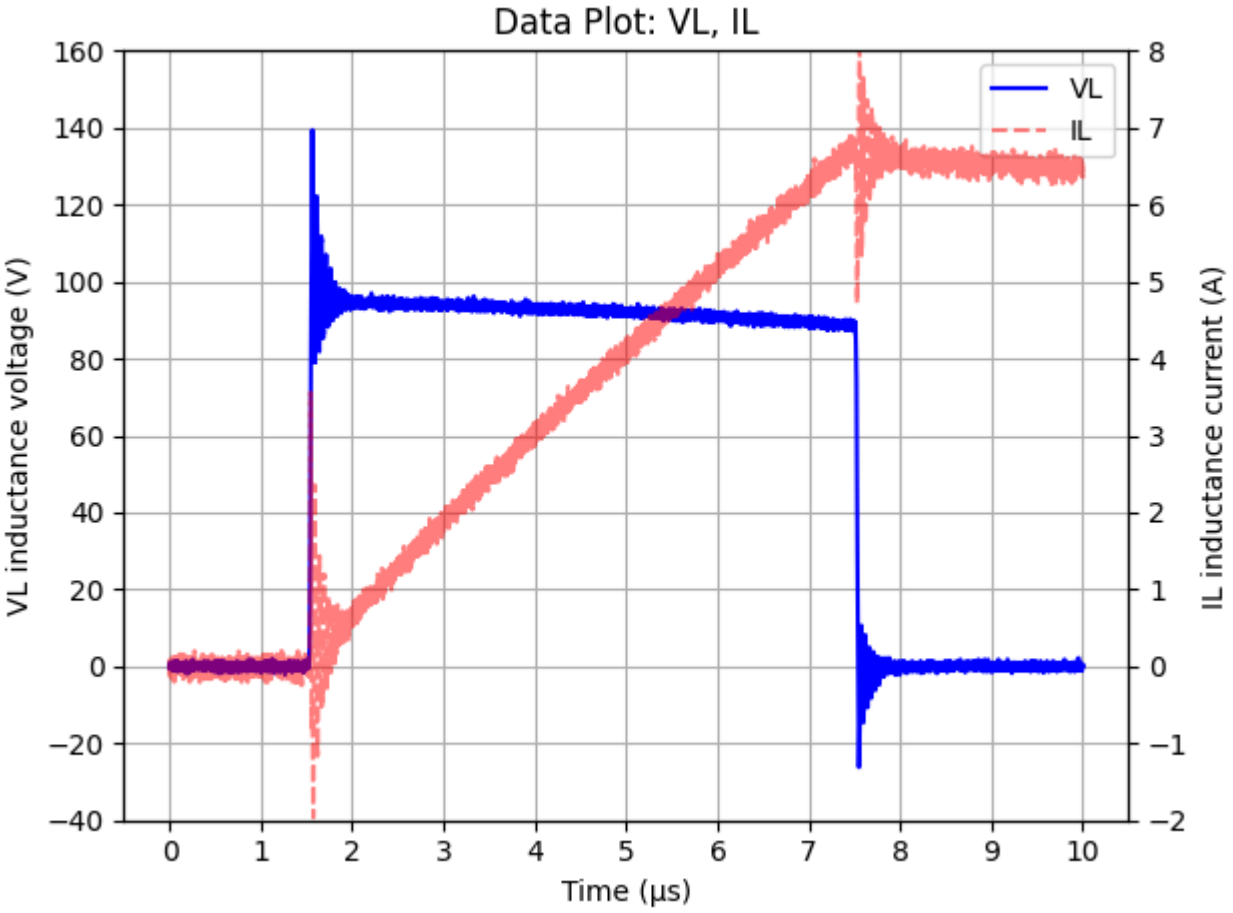
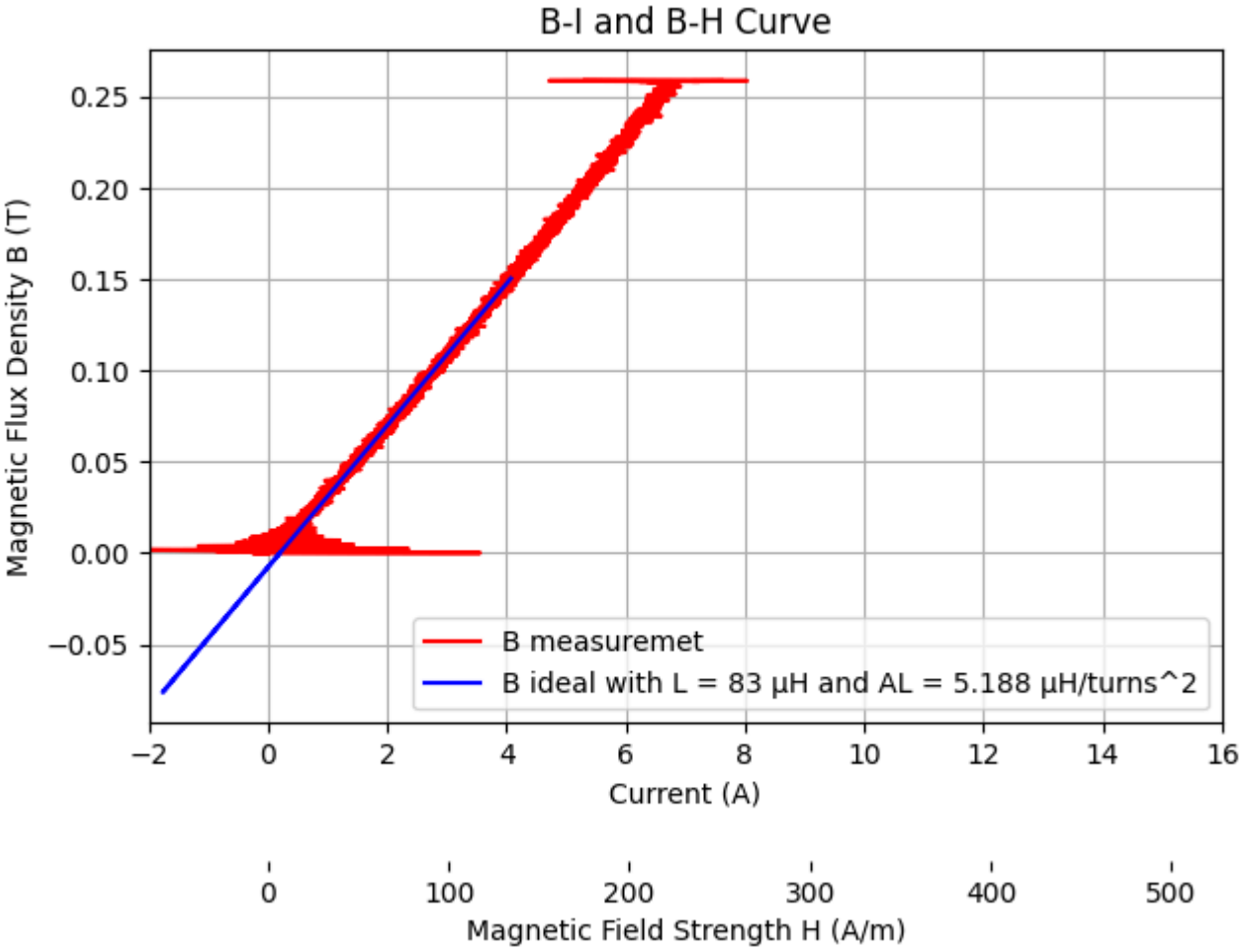


Figure 4: Oscilloscope screenshot, CH1 = V_L, CH2 = I_L

The plot shows the current and voltage data with zero correction at the beginning


Data Plot: VL, IL
VL
IL
VL inductance voltage (V)
IL inductance current (A)
Time (µs)
160
140
120
100
80
60
40
20
0
−20
−40
8
7
6
5
4
3
2
1
0
−1
−2
0
1
2
3
4
5
6
7
8
9
10



Estimating the real air gap

The characterized inductance is 83 µH, while the estimated value using the measured AL and the nominal air gap is 71 µH. Let's now estimate the actual air gap.

$$\mathrm{AL}_{MeasNoAirGap} = 13125 \ \ (\mathrm{nH/Turns2})$$

$$\mathrm{AL}_{MeasAirGap} = 83 \cdot \frac{1 \times 10^3}{(N)^2} = 83 \cdot \frac{1 \times 10^3}{(4)^2} \qquad = 5187.500 \ \ (\mathrm{nH/Turns2})$$

$$\begin{aligned} \mathrm{lg} &= 1 \times 10^3 \cdot \mu_0 \cdot \mathrm{Ae} \cdot 1 \times 10^9 \cdot \left(\frac{1}{\mathrm{AL}_{MeasAirGap}} - \frac{1}{\mathrm{AL}_{MeasNoAirGap}} \right) \\ &= 1 \times 10^3 \cdot 0.000 \cdot 0.001 \cdot 1 \times 10^9 \cdot \left(\frac{1}{5187.500} - \frac{1}{13125} \right) \\ &= 0.078 \ \ (\mathrm{mm}) \end{aligned}$$

$$\mathrm{lg}_{nom} = 0.100 \ \ (\mathrm{mm})$$

$$\mathrm{lg}_{error} = 100 \cdot \frac{\mathrm{lg} - \mathrm{lg}_{nom}}{\mathrm{lg}_{nom}} = 100 \cdot \frac{0.078 - 0.100}{0.100} \qquad = -21.740 \ \ (\backslash\%)$$

the air gap error of ~22% (100µm) is acceptable for kapton air gap without no control of the assembling pression

Conclusion

The primary magnetic inductance reference value for this project's transformer will be set to **83 µH**.

Lr Without air gap

In this test, we will use the resonant inductance choke with a PCB winding (8 turns), with no air gap is used.

The main objective is to compare the measured inductance with the datasheet value of this choke.

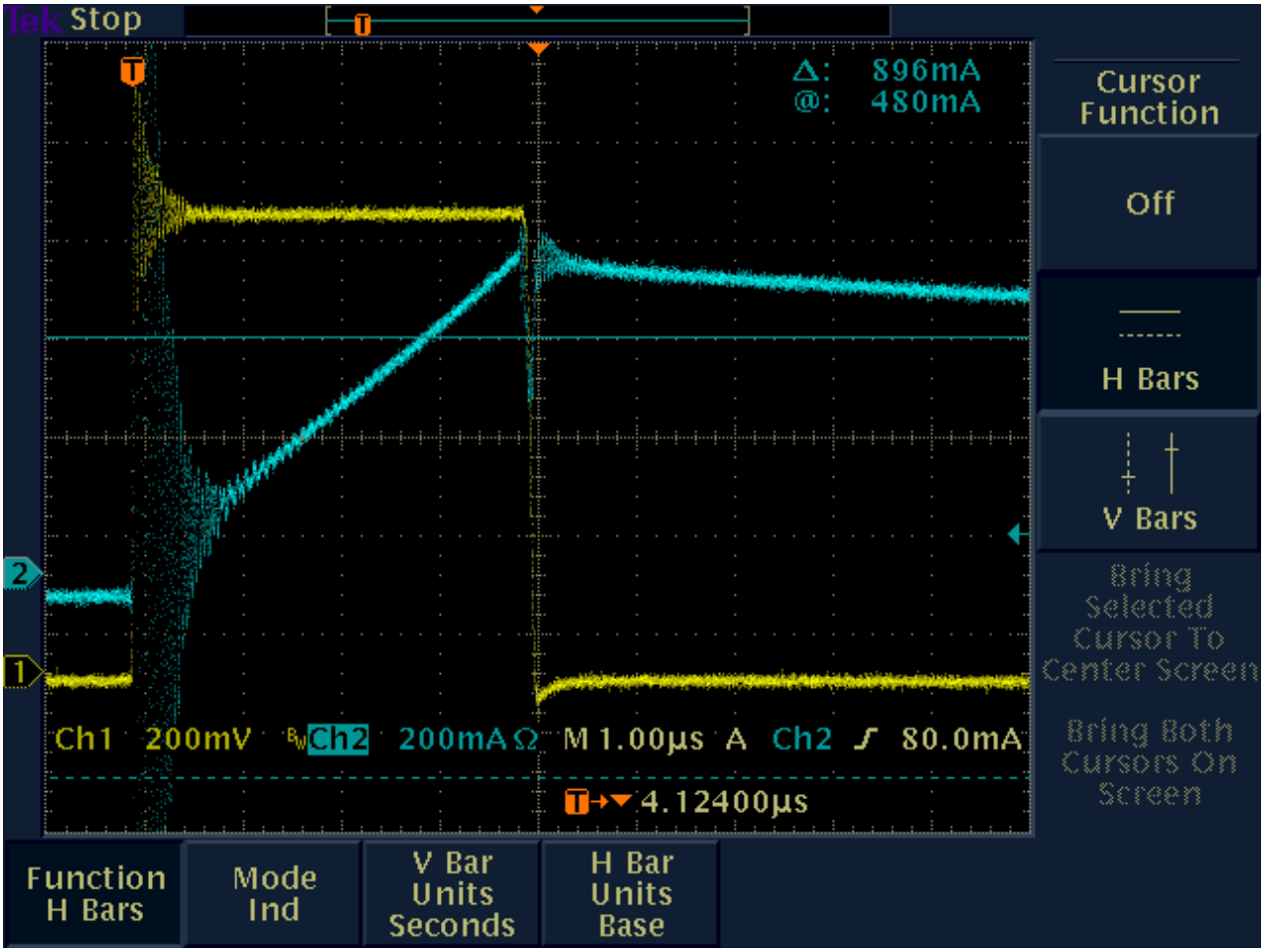
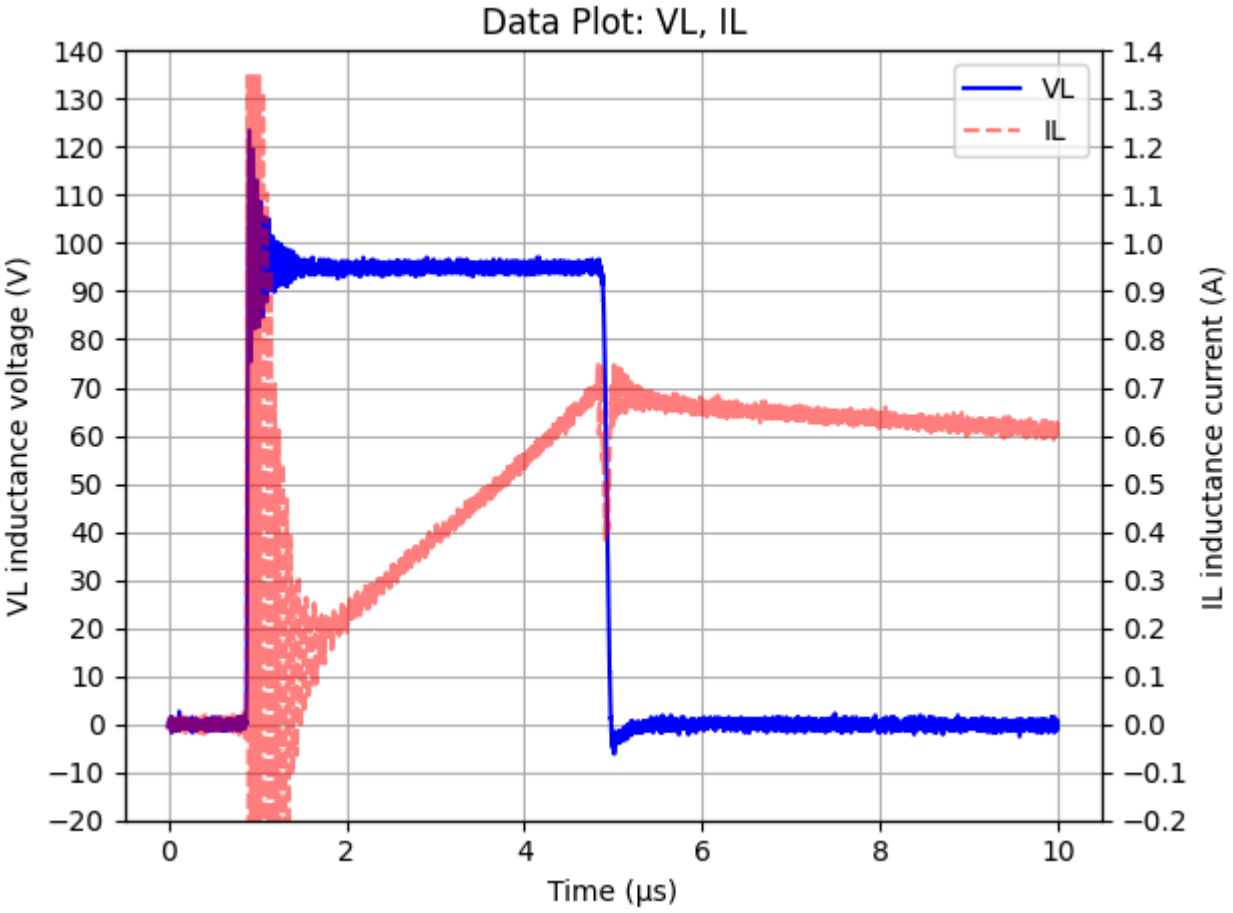


Figure 5: Oscilloscope screenshot, CH1 = V_L, CH2 = I_L

The plot shows the current and voltage data with zero correction at the beginning



See fromulas above.

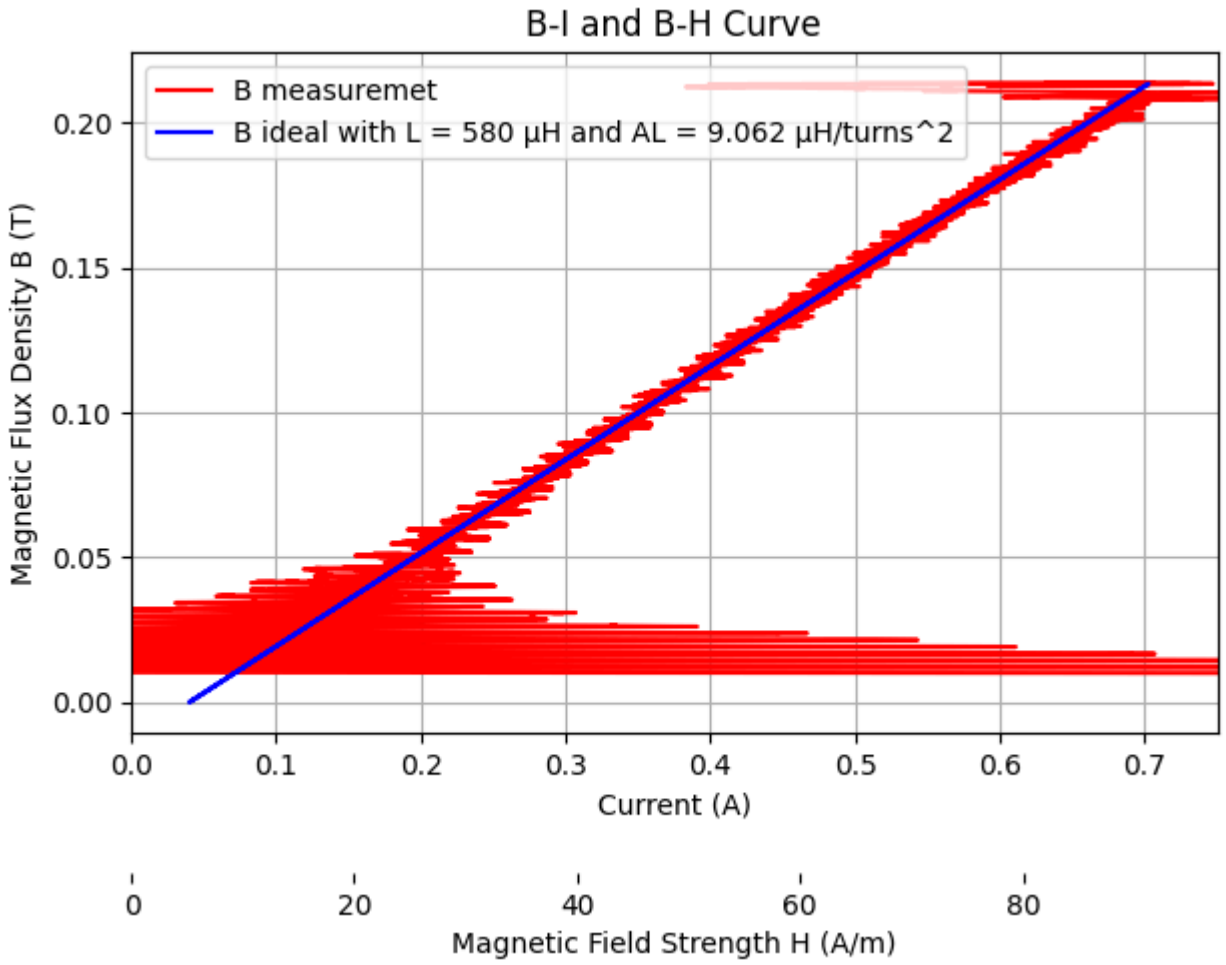
For more information, see the ELP 43/10/28 with ELP 43/10/28 N87 datasheet

$$A_e = 225 \ \ (\mathrm{mm2})$$

$$l_e = 61.600 \ \ (\mathrm{mm})$$

$$\mathrm{AL}_{nom} = 7300 \ \ (\mathrm{nH/N2})$$

idx_rise = trigger (vds, transition= "rise") idx_fall= trigger (vds, transition= "fall") #len(idx_rise), len(idx_fall)



Comparison between the measured value and the typical value provided in the datasheet:

$$\mathrm{Al}_{Measur} = 1 \times 10^{9} \cdot \mathrm{ALcal} = 1 \times 10^{9} \cdot 0.000 \qquad = 9062.500 \ \ (\mathrm{nH/turns2 : see\ the\ abov}$$

$$\mathrm{Al}_{nominal} = 7300 \ \ (\mathrm{nH/turns2 : see\ the\ datasheet})$$

$$\mathrm{Error}_{AL} = 100 \cdot \frac{\mathrm{Al}_{Measur} - \mathrm{Al}_{nominal}}{\mathrm{Al}_{nominal}} = 100 \cdot \frac{9062.500 - 7300}{7300} \qquad =$$

The measured error (24.1%) is in the the datasheet specification (+/-25%).

Estimating the inductance for a 800 µm (2x2×0.2 mm) air gap using the nominal AL value from the datasheet.

- 2x2×0.2 mm: We will use two layers, each with a thickness of 0.2 mm

$$\mathrm{Ae}_{mm2} = 225 \ \ (\text{see the datasheet})$$

$$\mathrm{lg}_{mm} = 0.2 \cdot 2 \cdot 2 \qquad = 0.800 \ \ (\mathrm{m}$$

$$\mathrm{lg} = 1 \times 10^{-3} \cdot \mathrm{lg}_{mm} = 1 \times 10^{-3} \cdot 0.800 \qquad = 0.001 \ \ (0.10\text{e-}3 \ \ \text{Total gap 2 * 0.05 m}$$

$$\mathrm{Al}_{nom} = \mathrm{Al}_{nominal} \cdot 1 \times 10^{-9} = 7300 \cdot 1 \times 10^{-9} \qquad = 0.$$

$$\begin{aligned} \mathrm{AL}_{gap_{nH1}} &= \frac{1 \times 10^{9}}{\left(\frac{1}{\mathrm{Al}_{nom}}\right) + \left(\frac{\mathrm{lg}}{\mu_0 \cdot \mathrm{Ae}_{mm2} \cdot 1 \times 10^{-6}}\right)} \\ &= \frac{1 \times 10^{9}}{\left(\frac{1}{0.000}\right) + \left(\frac{0.001}{0.000 \cdot 225 \cdot 1 \times 10^{-6}}\right)} \\ &= 337.108 \ \ (\mathrm{nH} \ / \ \text{turns 2}) \end{aligned}$$

$$\mathrm{Lnew1} = 1 \times 10^{-3} \cdot (N)^2 \cdot \mathrm{AL}_{gap_{nH1}} = 1 \times 10^{-3} \cdot (8)^2 \cdot 337.108 \qquad = 21.575 \ \ (\mu$$

Estimating the inductance for a 800 µm (2x2×0.2 mm) air gap using the nominal AL value from the measurement.

$$\begin{aligned} \mathrm{AL}_{gap_{nH2}} &= \frac{1 \times 10^{9}}{\left(\frac{1 \times 10^{9}}{\mathrm{Al}_{Measur}}\right) + \left(\frac{\mathrm{lg}}{\mu_0 \cdot \mathrm{Ae}_{mm2} \cdot 1 \times 10^{-6}}\right)} \\ &= \frac{1 \times 10^{9}}{\left(\frac{1 \times 10^{9}}{9062.500}\right) + \left(\frac{0.001}{0.000 \cdot 225 \cdot 1 \times 10^{-6}}\right)} \\ &= 340.163 \ \ (\mathrm{nH} \ / \ \text{turns 2}) \end{aligned}$$

$$\mathrm{Lnew2} = 1 \times 10^{-3} \cdot (N)^2 \cdot \mathrm{AL}_{gap_{nH2}} = 1 \times 10^{-3} \cdot (8)^2 \cdot 340.163 \qquad = 21.770 \ \ (\mu\mathrm{H})$$

$$\mathrm{Error} = 100 \cdot \frac{\mathrm{AL}_{gap_{nH2}} - \mathrm{AL}_{gap_{nH1}}}{\mathrm{AL}_{gap_{nH1}}} = 100 \cdot \frac{340.163 - 337.108}{337.108} \qquad = 0.906 \ \ (\backslash\%)$$

The dispersion error with the air gap is only 1%, making it negligible compared to the initial 24% dispersion without an air gap.

This result comes from the fact that the air gap is relatively large (nearly 1 mm).

Lr with air gap

We will use plastic layers with a thickness of 0.2 mm to form the air gap, resulting in a total air gap of 2x2×200 µm = 800 µm (two layers).

The image below presents the thickness measurement of the 0.2 mm layers using two techniques, where the 5-layers method provides higher accuracy.



Figure 6: Plastic insulation for magnetic air gap: One Layer, thickness 200µm



Figure 7: Plastic insulation for magnetic air gap: 5 Layers, thickness 1mm

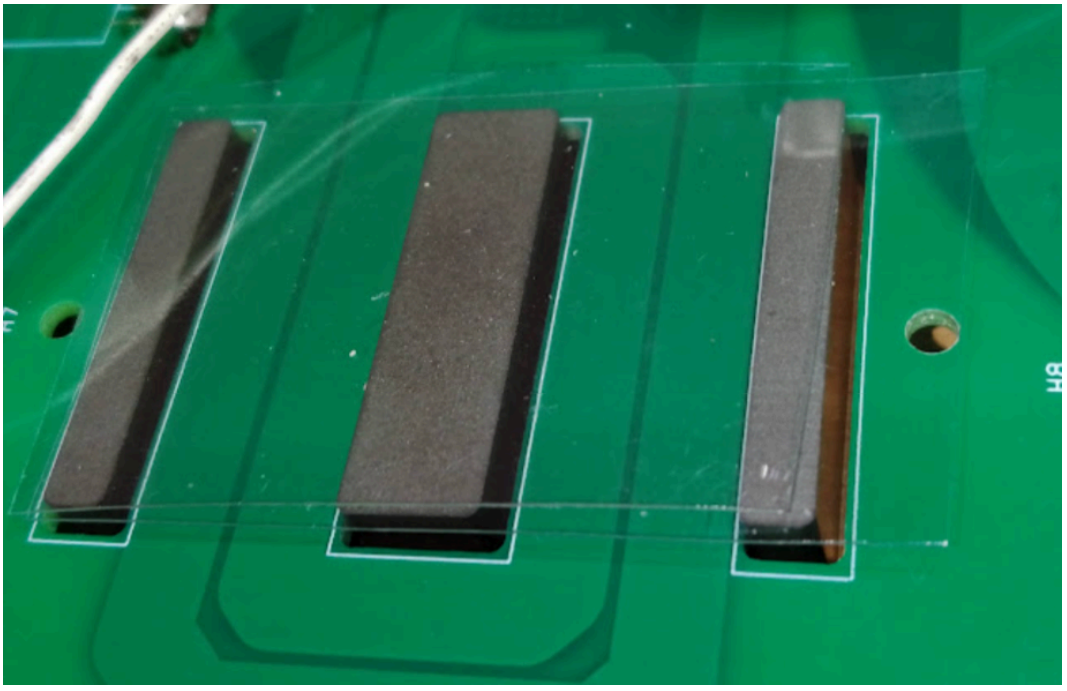


Figure 8: two 0.2 mm plastic layers as Lr air gap, total air gap 0.8mm

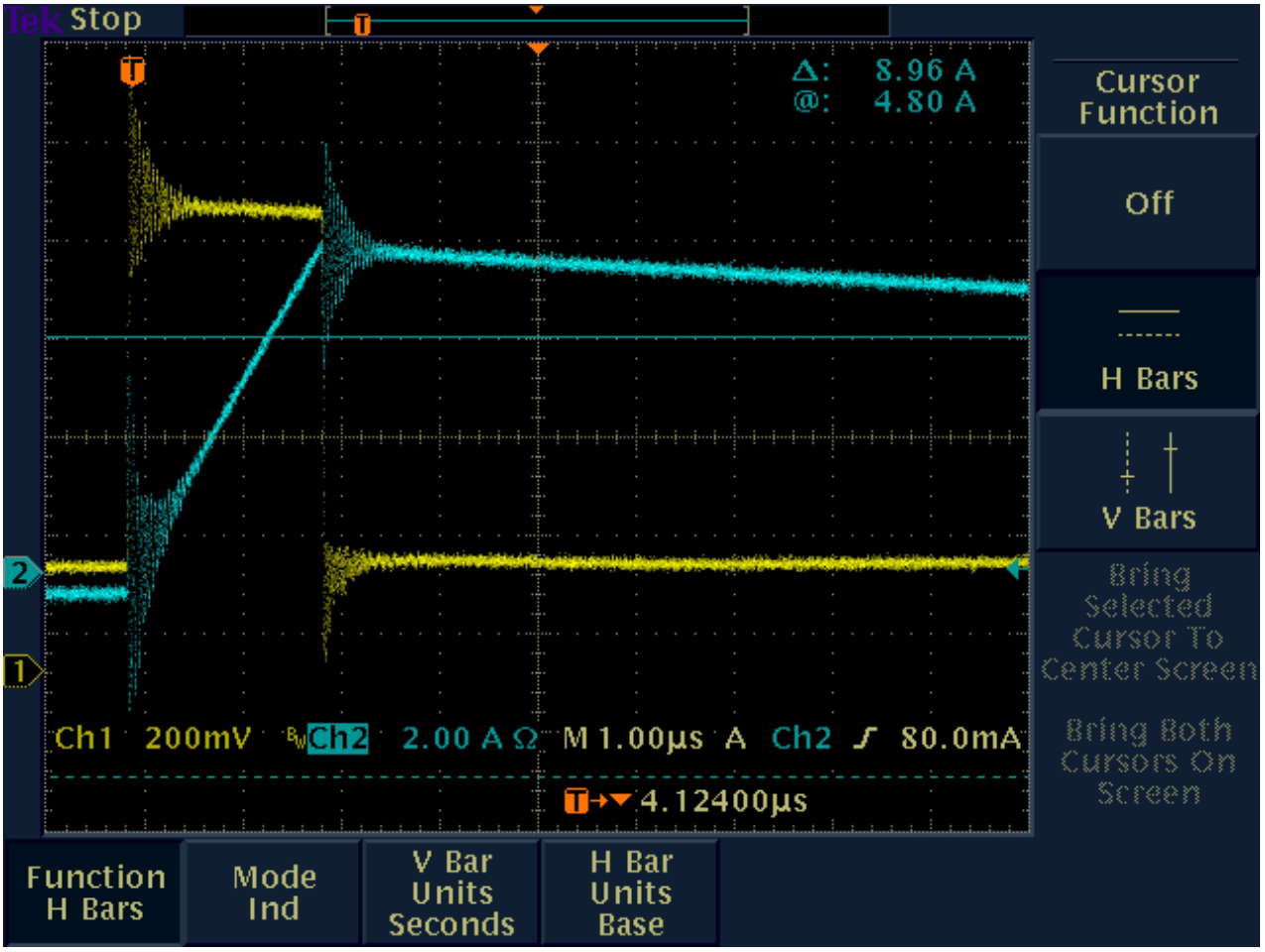
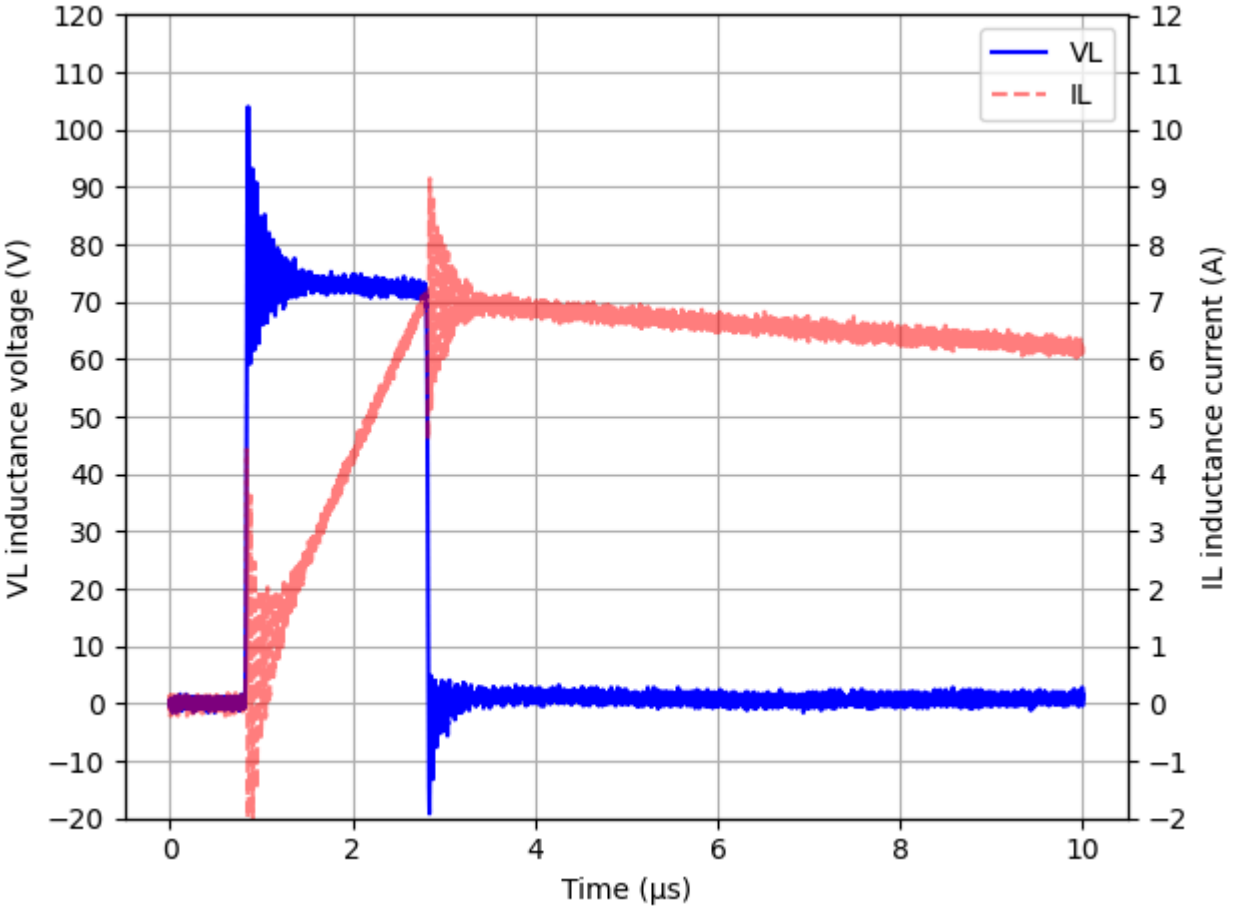
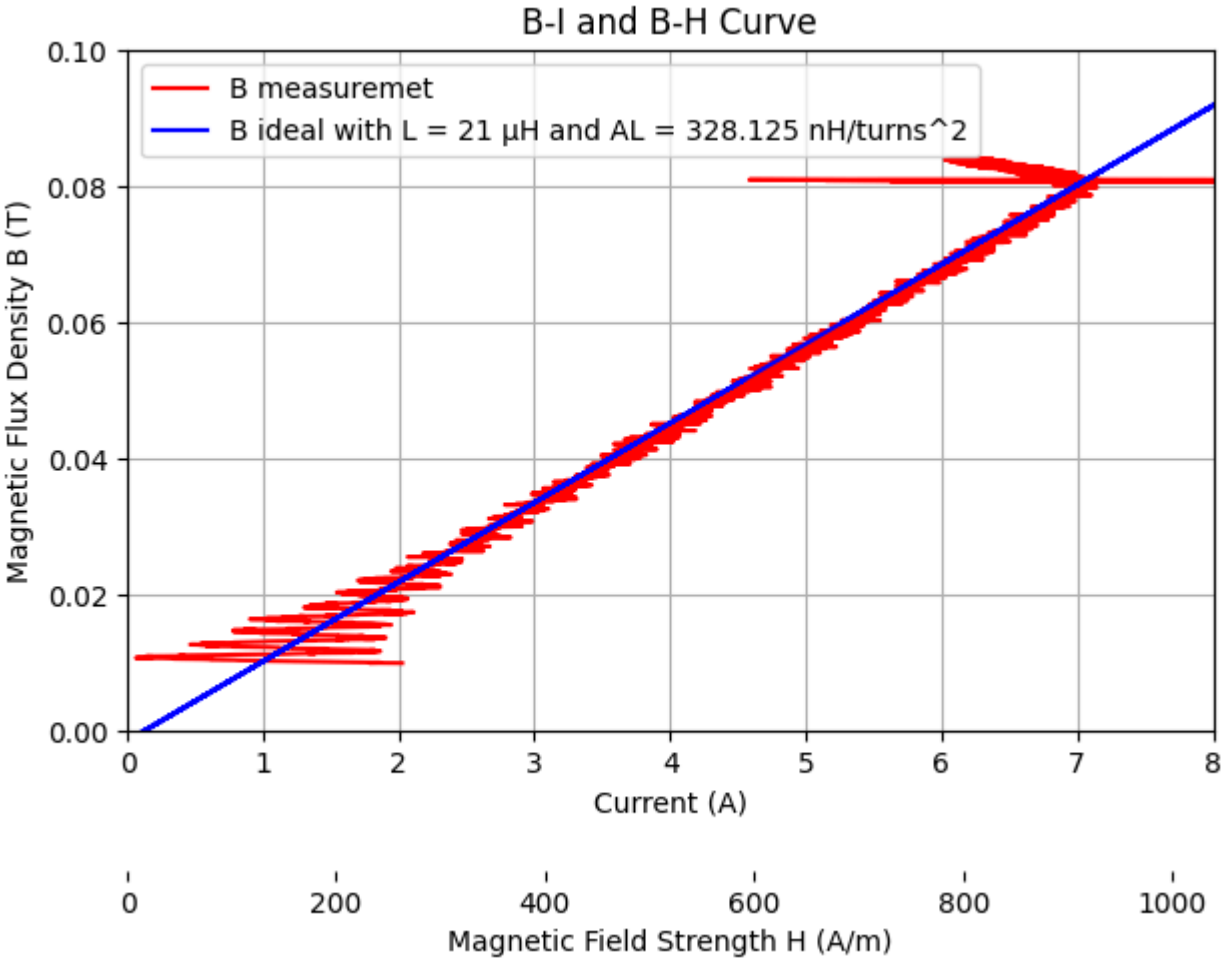


Figure 9: Oscilloscope screenshot, CH1 = V_L, CH2 = I_L

The plot shows the current and voltage data with zero correction at the beginning



idx_rise = trigger (vds, transition= "rise") idx_fall= trigger (vds, transition= "fall") #len(idx_rise), len(idx_fall)



Estimating the real air gap

The characterized inductance is 21 µH, while the estimated value using the measured AL and the nominal air gap is 21.77µH. Let's now estimate the actual air gap.

$$\mathrm{AL}_{MeasNoAirGap} = \frac{580 \times 10^3}{(N)^2} = \frac{580 \times 10^3}{(8)^2} \qquad = 9062.500 \ \ (\mathrm{nH/Turns2})$$

$$\mathrm{AL}_{MeasAirGap} = 21 \cdot \frac{1 \times 10^3}{(N)^2} = 21 \cdot \frac{1 \times 10^3}{(8)^2} \qquad = 328.125 \ \ (\mathrm{nH/Turns2})$$

$$\begin{aligned} \mathrm{lg} &= 1 \times 10^3 \cdot \mu_0 \cdot \mathrm{Ae} \cdot 1 \times 10^9 \cdot \left(\frac{1}{\mathrm{AL}_{MeasAirGap}} - \frac{1}{\mathrm{AL}_{MeasNoAirGap}} \right) \\ &= 1 \times 10^3 \cdot 0.000 \cdot 0.000 \cdot 1 \times 10^9 \cdot \left(\frac{1}{328.125} - \frac{1}{9062.500} \right) \\ &= 0.830 \ \ (\mathrm{mm}) \end{aligned}$$

$$\mathrm{lg}_{nom} = 0.800 \ \ (\mathrm{mm})$$

$$\mathrm{lg}_{error} = 100 \cdot \frac{\mathrm{lg} - \mathrm{lg}_{nom}}{\mathrm{lg}_{nom}} = 100 \cdot \frac{0.830 - 0.800}{0.800} \qquad = 3.812 \ \ (\backslash\%)$$

the air gap error of ~4% (800µm) is acceptable for plastic air gap without no control of the assembling pression

The final result of Lr, Lm

The double-pulse method is used as the reference technique for characterizing the resonant inductance (Lr) and magnetizing inductance (Lm). Additional complementary methods are also presented in this document. The reference values are defined as follows:

- **Lr = 21 µH** (see double-pulse test for Lr with air gap)
- **Lm = 83 µH** (see double-pulse test for Lm with air gap)

Resonant-circuit-based measurement method

Setup

For this test, we will characterize the inductance using the LC resonance method.

The setup and the key formulas required for the characterization are shown below.

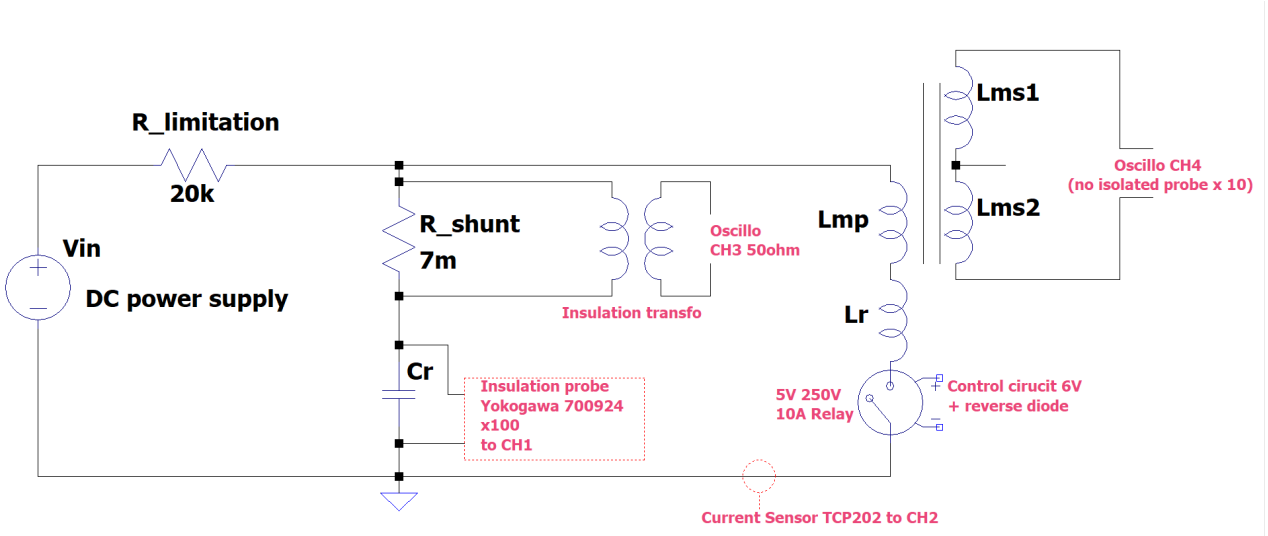


Figure 10: Overview of the setup (schematic)



Figure 11: Overview of the setup (Image)



Figure 12: Insulation transformator

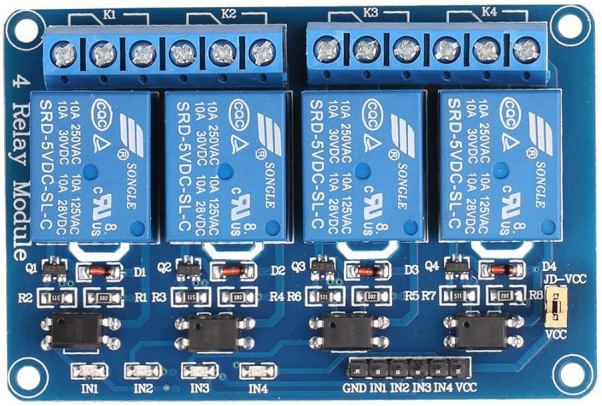


Figure 13: Used relay

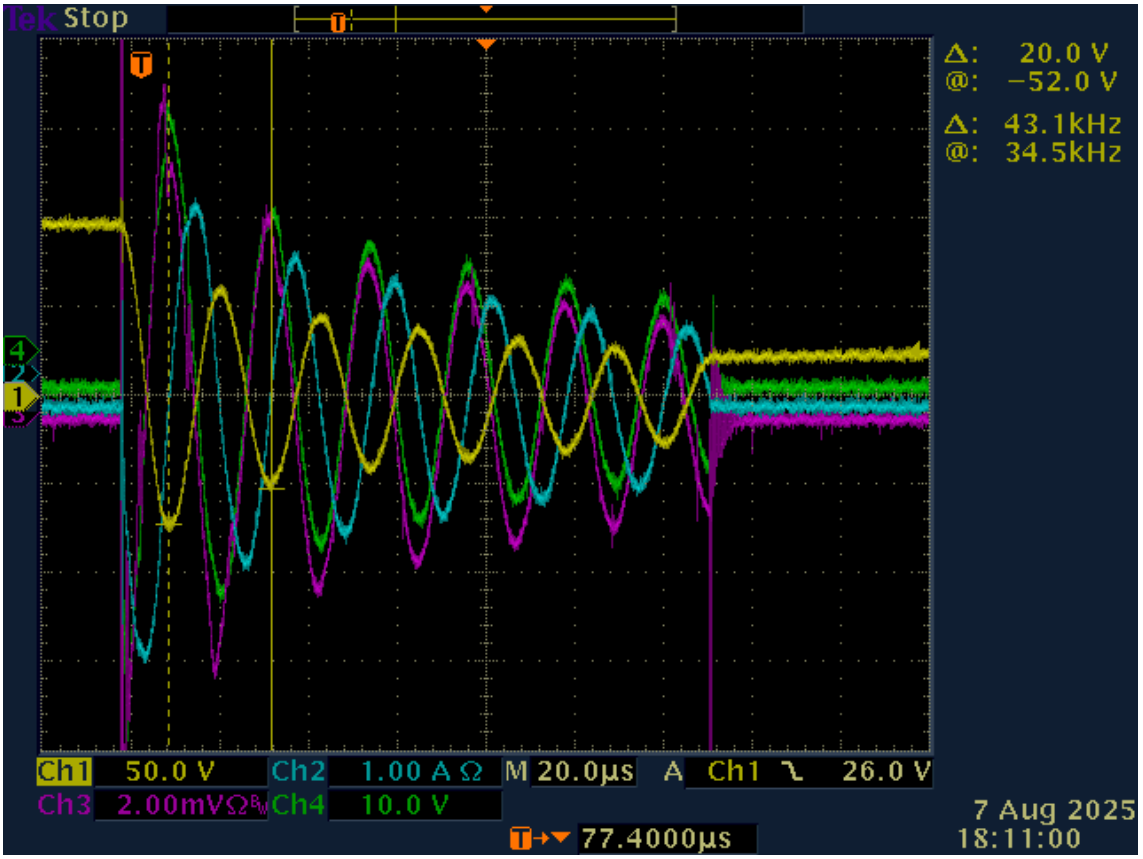


Figure 14: Oscilloscope screenshot example

FORMULAS

In an ideal **LC** resonance circuit, the peak current in the inductor can be controlled by two factors: the capacitor value and its initial charging voltage, meaning the total initial energy stored in the capacitor.

The goal is to estimate the inductance using theoretical calculation or small-signal measurements, and then select the capacitor **C** and the initial voltage **V0** to achieve the desired peak current and resonance frequency.

Both current and frequency are key for inductance characterization. The formulas are shown below:

Ideal LC:

The energy stored in the capacitor and in the inductor at their respective maximum energy points:

$$E_C = \frac{1}{2} \cdot C \cdot (V_0)^2$$

$$E_L = \frac{1}{2} \cdot L \cdot (I_{peak})^2$$

Since no resistor, so:

$$E_L = E_L$$

So, we can calculate the peak current using:

$$I_{peak} = V_0 \cdot \sqrt{\frac{C}{L}}$$

And the frequency will be approximately equal to the ideal resonance frequency given by:

$$F_{res0} = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

With resistor (dumping): LCR

Dumping factor:

$$m = \left(\frac{R}{2}\right) \cdot \sqrt{\frac{C}{L}}$$

Alpha:

$$\alpha = \omega_0 \cdot m$$

Dumping frequency:

$$F_{resd} = F_{res0} \cdot \sqrt{1 - m \cdot m}$$

$$\omega_d = \omega_0 \cdot \sqrt{1 - m \cdot m}$$

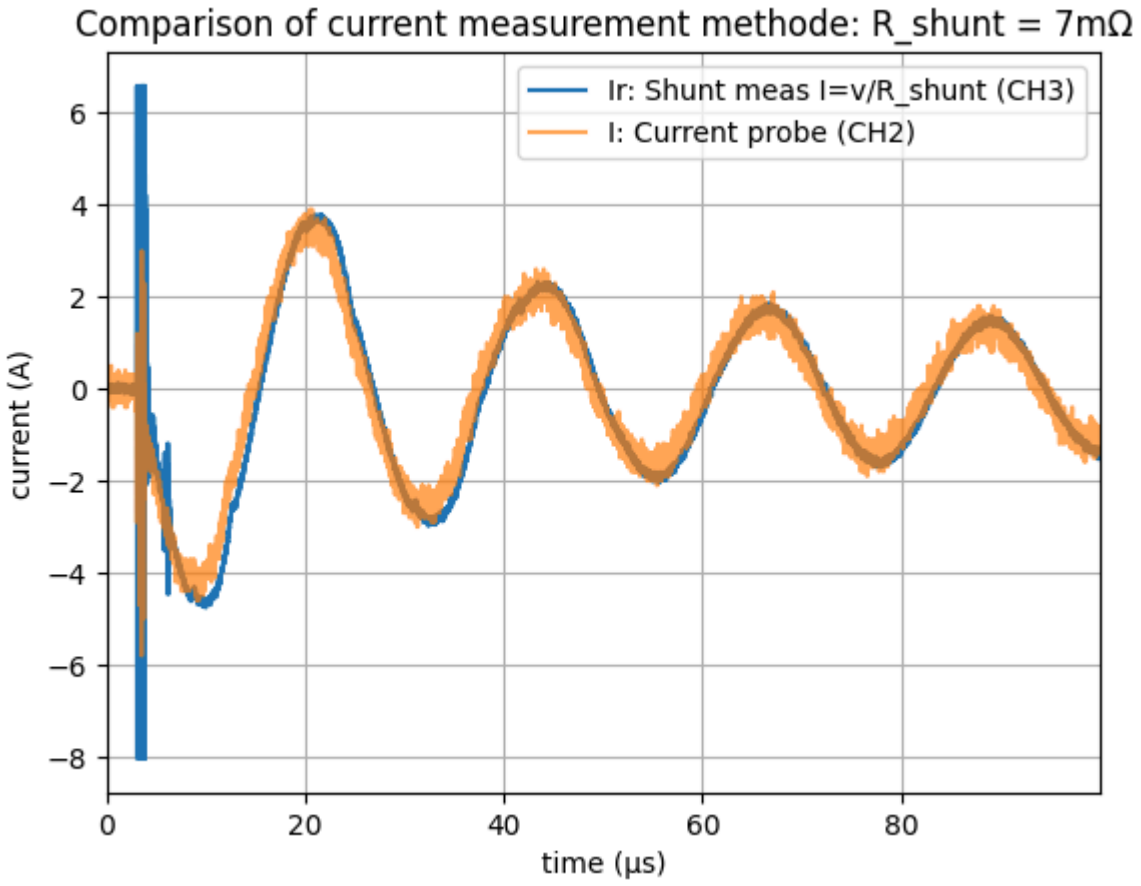
And the capacitor voltage :

$$V_0 \cdot \exp((-\alpha) \cdot t) \cdot \sin(\omega_d \cdot t - \text{phase})$$

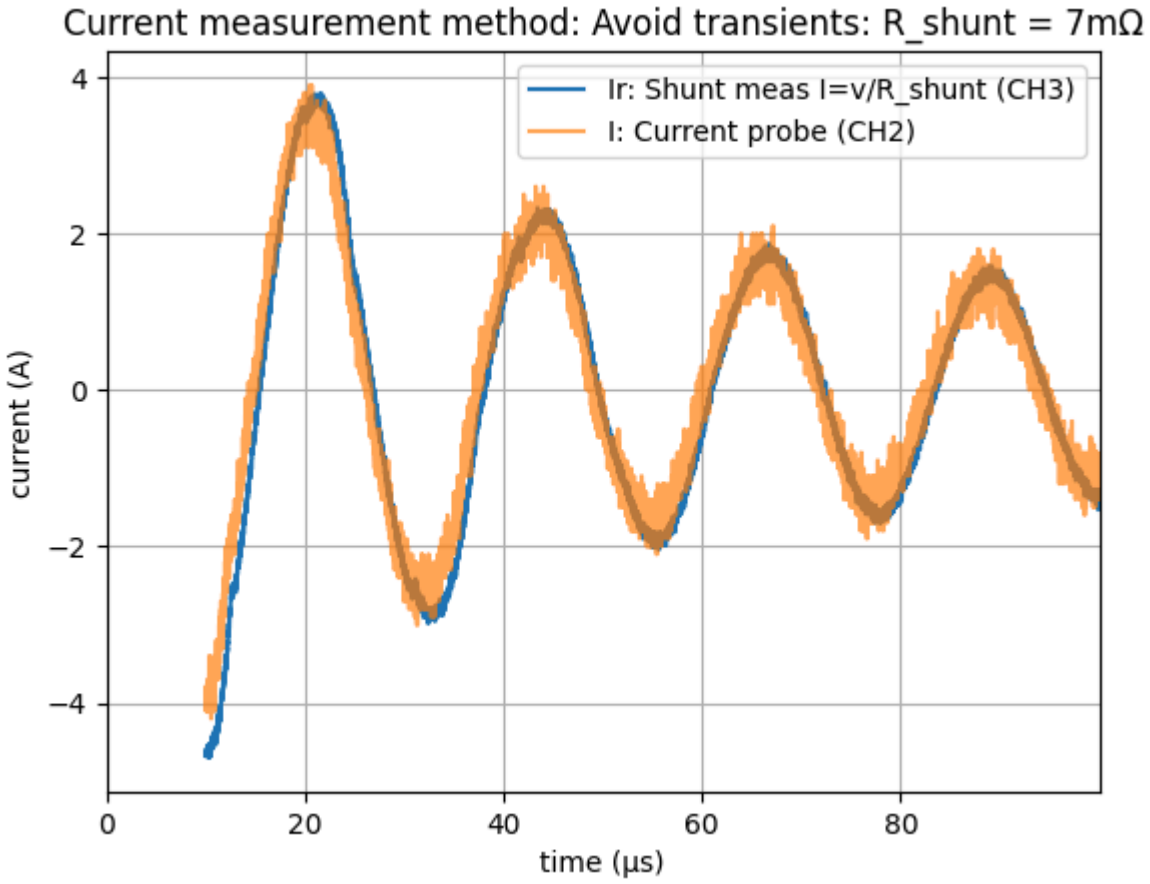
Current measurement methodes

In this sub-section, we will take advantage of the test to verify a few current measurement methods.

If you want only inductance caracterisation, go to the next title (Lr+Lm).

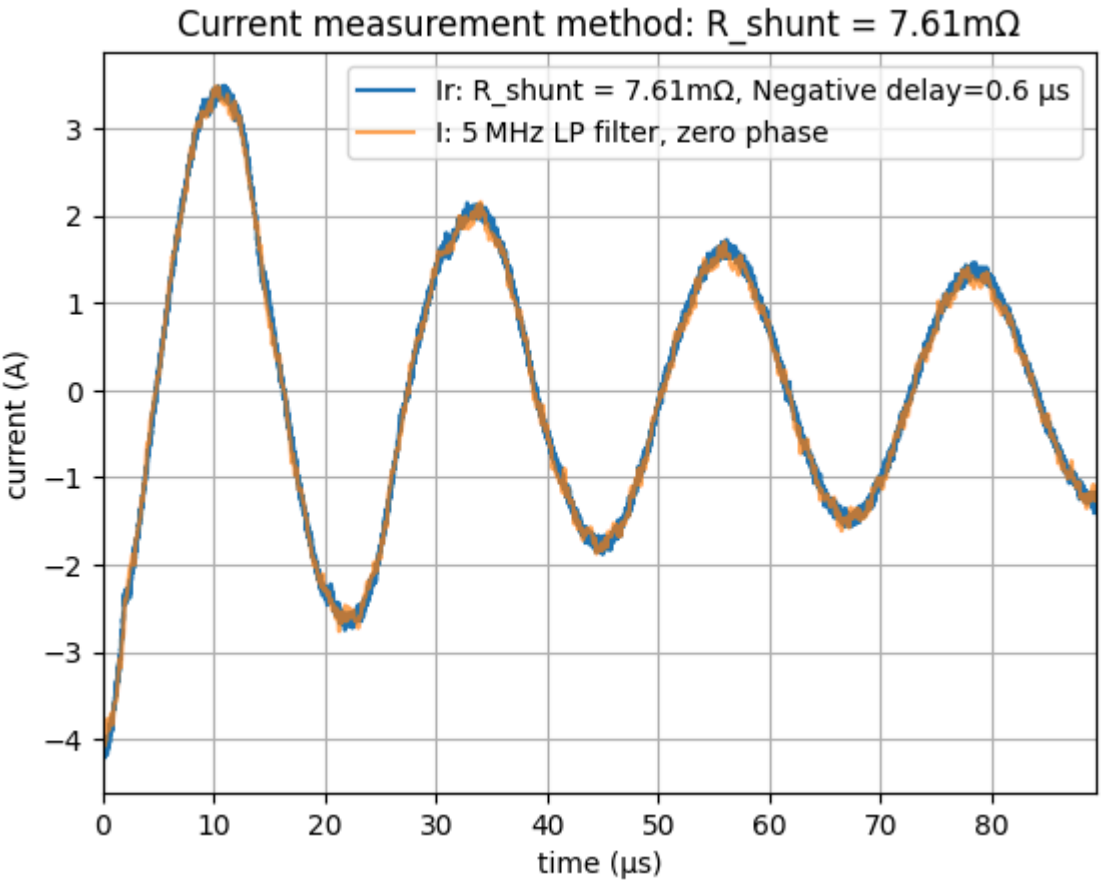


Above is the full signal, and below is only 90% of the signal to avoid the initial transient phase.



Below is some advanced preprocessing:

for the TCP202 current probe, we apply a non-causal zero-phase filter with a 5 MHz cutoff frequency, and for the shunt measurement, we introduce a −0.6 µs delay to compensate for the phase shift introduced by the signal transformer.

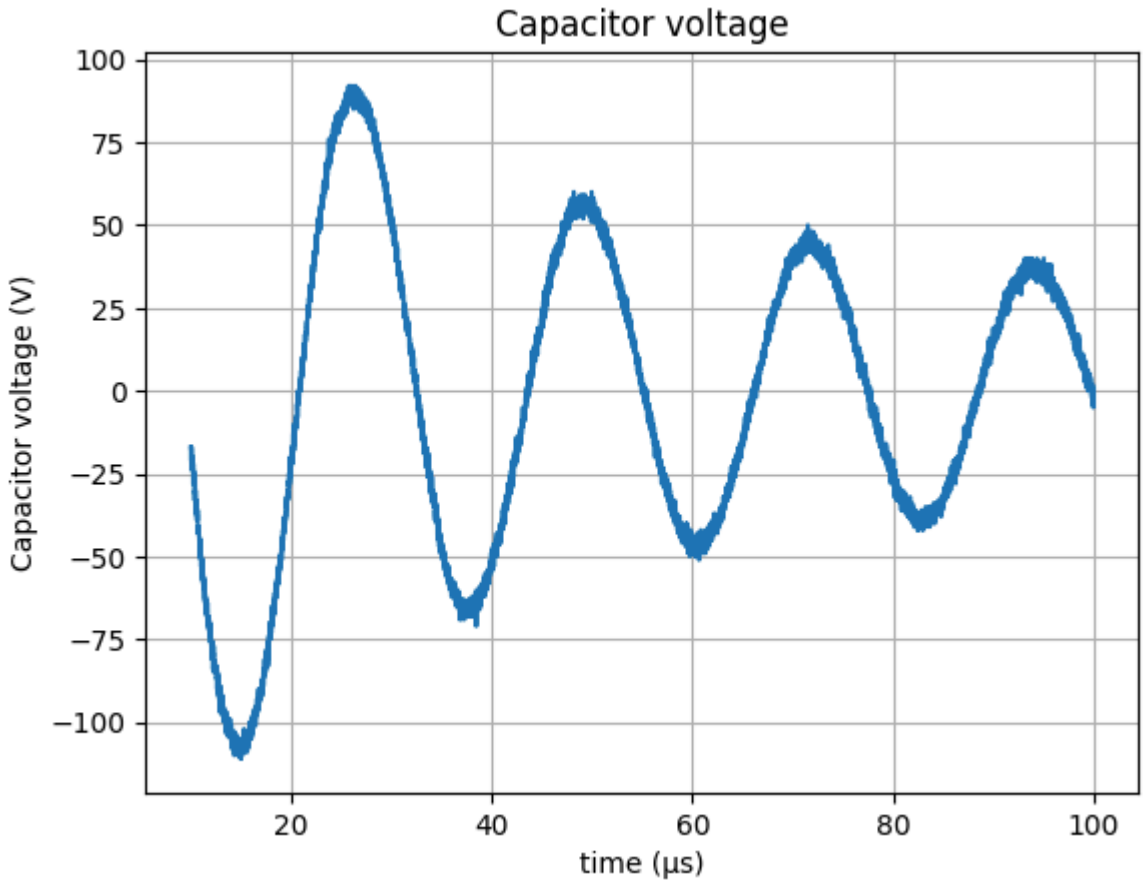


Conclusion

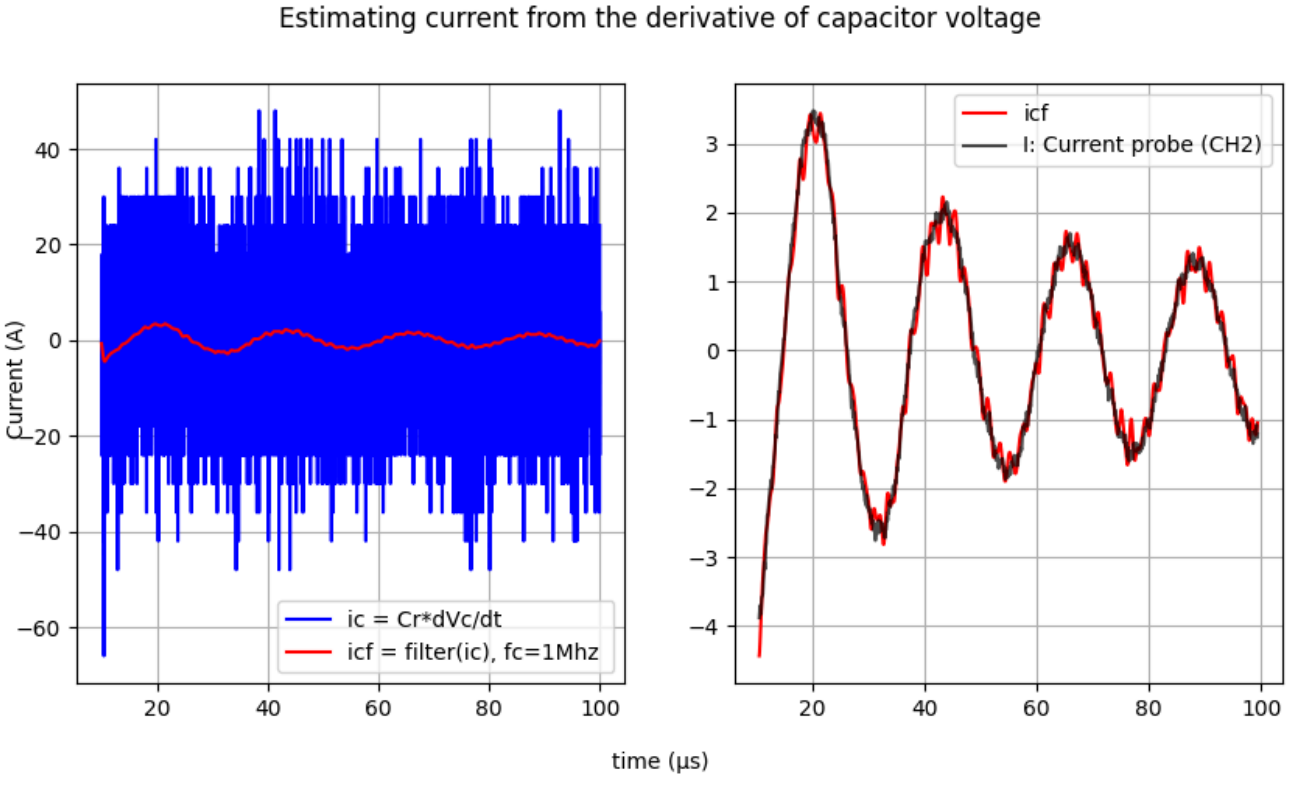
A shunt combined with a signal transformer can be used as a fast current probe, but this setup introduces a relatively large delay of about 600 ns.

Using capacitor voltage to estimate the current

below we will check this methode:



Below the comparaison between the probe current (TCP202) and the estimated current using the capacitor voltage derivation



Conclusion

Capacitor-voltage derivation is a valid method when combined with preprocessing using a non-causal zero-phase filter, as illustrated above. The raw derivative signal is unusable (see the blue curve).

However, using a 1 MHz low-pass filter limits the current-measurement bandwidth. Therefore, this approach can serve as an alternative when direct measurement is not feasible.

Lr+Lm

In this test, we will use the damping formulas presented in the setup section. The idea is to adjust L, C, and R until we obtain the best fit between the measured curve and the theoretical response of the damped LRC circuit, allowing us to determine the inductance L.

As a complementary method, we will also compute the resonance frequency to extract the characterized value of L.

Initial value

$$\mathrm{Lm}_{uH} = 83 \ \ (\mu\mathrm{H}) \qquad \mathrm{Lr}_{uH} = 21 \ \ (\mu\mathrm{H}) \qquad \mathrm{Cr}_{nF} = 120 \ \ (\mathrm{nF})$$

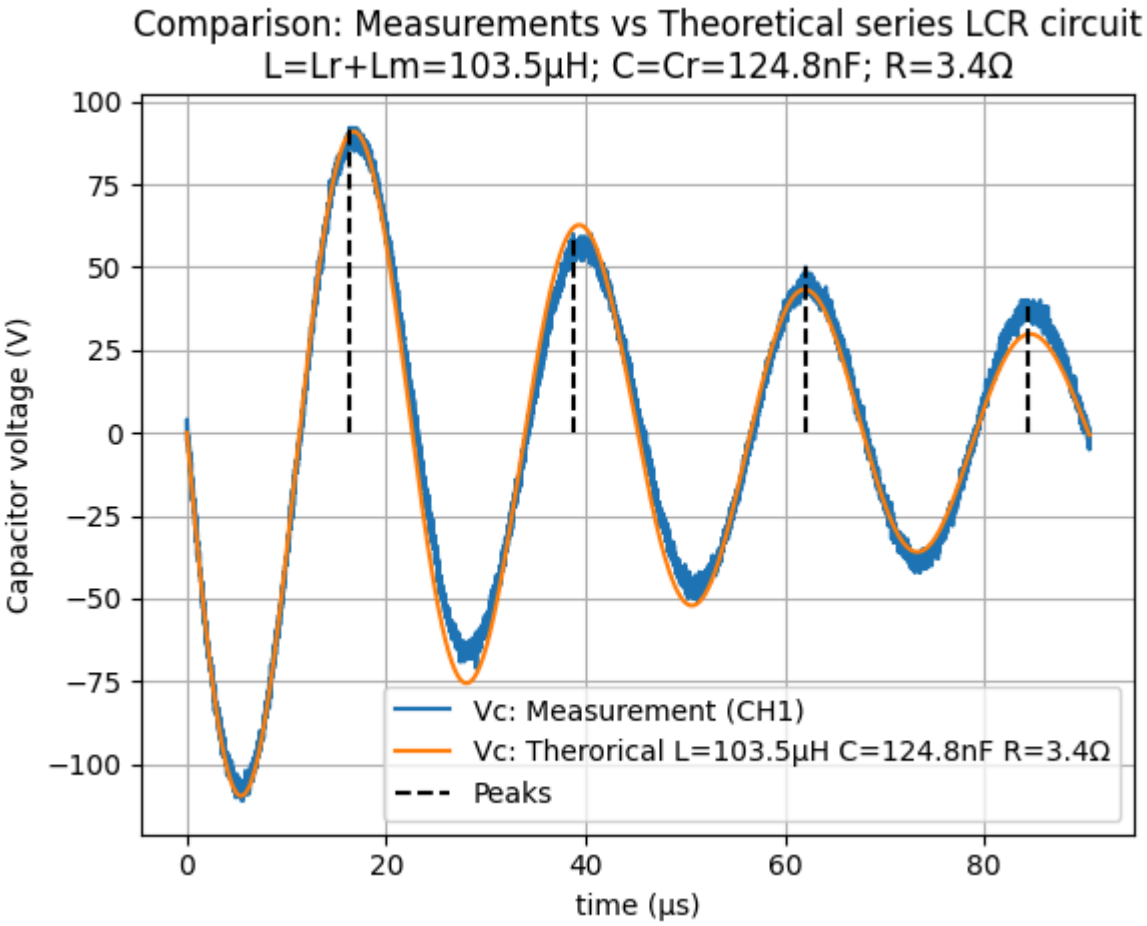
Tuned values

$$\mathrm{Lm}_{uH} = 79 \ \ (\mu\mathrm{H}) \qquad \mathrm{Lr}_{uH} = 24.5 \ \ (\mu\mathrm{H}) \qquad \mathrm{Cr}_{nF} = 124.8 \ \ (\mathrm{nF})$$

$$R = 3.4 \ \ (\Omega)$$

Total inductance

$$L_{uH} = \mathrm{Lm}_{uH} + \mathrm{Lr}_{uH} = 79 + 24.5 \qquad = 103.5 \ \ (\mathrm{uH})$$



`Resonant frequency estimated from the average peak interval (see above): Fr = 44.08 kHz`

Theoretical resonant frequency calculated from the L and C values:

$$C = \mathrm{Cr}_{nF} \cdot 1 \times 10^{-9} = 124.8000000000 \cdot 1 \times 10^{-9} \qquad = 0.0000001248$$

$$\mathrm{Fr} = \frac{1 \times 10^{-3}}{2 \cdot \pi \cdot \sqrt{L \cdot \mathrm{Cr}_{nF} \cdot 1 \times 10^{-9}}} = \frac{1 \times 10^{-3}}{2 \cdot 3.142 \cdot \sqrt{0.000 \cdot 124.800 \cdot 1 \times 10^{-9}}} \qquad = 44.284 \ \ (\mathrm{khz})$$

$$m = \left(\frac{R}{2}\right) \cdot \sqrt{\frac{C}{L}} = \left(\frac{3.400}{2}\right) \cdot \sqrt{\frac{0.000}{0.000}} \qquad = 0.059 \ \ (\text{damping factor})$$

$$\mathrm{FreqD}_{theo} = \mathrm{Fr} \cdot \sqrt{1 - m \cdot m} = 44.284 \cdot \sqrt{1 - 0.059 \cdot 0.059} \qquad = 44.206 \ \ (\text{kHz, damped frequency})$$

$$\mathrm{FreqD}_{meas} = 44.080$$

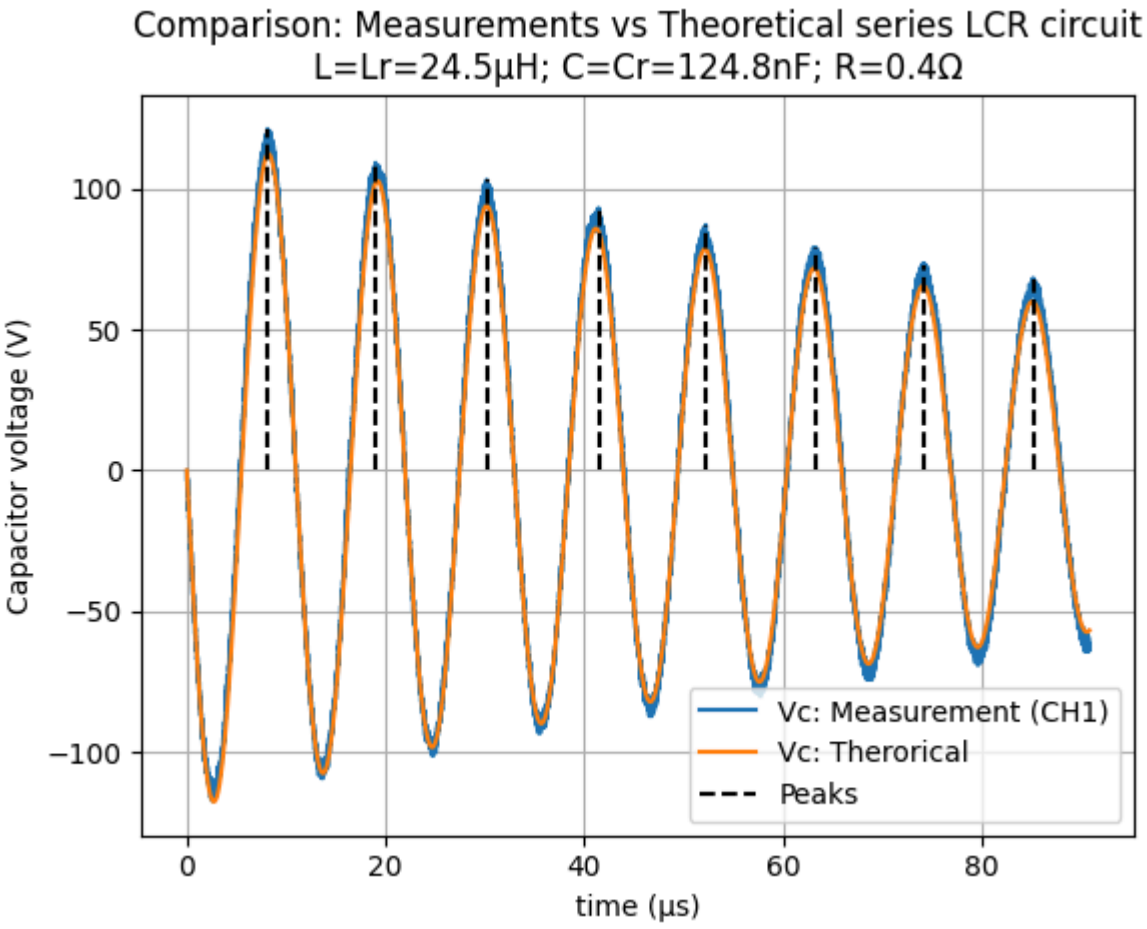
$$\mathrm{Freq}_{Error} = 100 \cdot \frac{\mathrm{FreqD}_{meas} - \mathrm{FreqD}_{theo}}{\mathrm{FreqD}_{theo}} = 100 \cdot \frac{44.080 - 44.206}{44.206} \qquad = -0.286 \ \ (\backslash\%)$$

The error between the tuned resonant LRC frequency and the average measured value is 0.3%, which indicates a very good estimation.

Lr

Tuned values

$$\mathrm{Lr}_{uH} = 24.500 \ \ (\mu\mathrm{H}) \qquad \mathrm{Cr}_{nF} = 124.800 \ \ (\mathrm{nF}) \qquad R = 0.400 \ \ (\Omega)$$



`Resonant frequency estimated from the average peak interval (see above): Fr = 90.77 kHz`

Theoretical resonant frequency calculated from the L and C values:

$$C = \mathrm{Cr}_{nF} \cdot 1 \times 10^{-9} = 124.800000000 \cdot 1 \times 10^{-9} \qquad = 0.000000125$$

$$\mathrm{Fr} = \frac{1 \times 10^{-3}}{2 \cdot \pi \cdot \sqrt{L \cdot \mathrm{Cr}_{nF} \cdot 1 \times 10^{-9}}} = \frac{1 \times 10^{-3}}{2 \cdot 3.142 \cdot \sqrt{0.000 \cdot 124.800 \cdot 1 \times 10^{-9}}} \qquad = 91.019 \ \ (\mathrm{khz})$$

$$m = \left(\frac{R}{2}\right) \cdot \sqrt{\frac{C}{L}} = \left(\frac{0.4000}{2}\right) \cdot \sqrt{\frac{0.0000}{0.0000}} \qquad = 0.0143 \ \ (\text{damping factor})$$

$$\mathrm{FreqD}_{theo} = \mathrm{Fr} \cdot \sqrt{1 - m \cdot m} = 91.0185 \cdot \sqrt{1 - 0.0143 \cdot 0.0143} \qquad = 91.0093 \ \ (\mathrm{kHz}, \text{ damped frequency})$$

The damped frequency erorr:

$$\mathrm{FreqD}_{meas} = 90.770$$

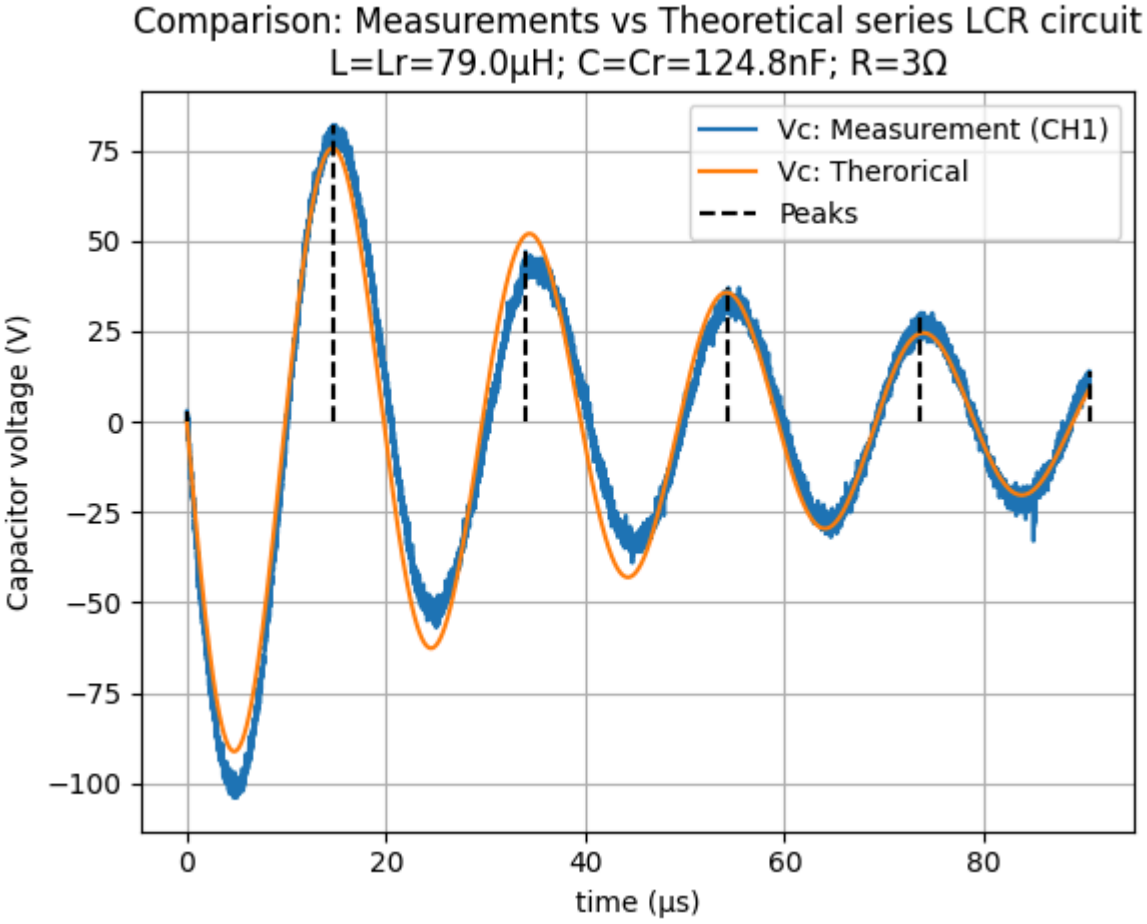
$$\mathrm{Freq}_{Error} = 100 \cdot \frac{\mathrm{FreqD}_{meas} - \mathrm{FreqD}_{theo}}{\mathrm{FreqD}_{theo}} = 100 \cdot \frac{90.770 - 91.009}{91.009} \qquad = -0.263 \ \ (\backslash\%)$$

The error between the tuned resonant LRC frequency and the average measured value is 0.3%, which indicates a very good estimation.

Lm

Tuned values

$$\mathrm{Lm}_{uH} = 79 \ \ (\mu\mathrm{H}) \qquad \mathrm{Cr}_{nF} = 124.800 \ \ (\mathrm{nF}) \qquad R = 3 \ \ (\Omega)$$




```
Resonant frequency estimated from the average peak interval (see above): Fr = 50.84 kHz
```

Theoretical resonant frequency calculated from the L and C values:

$$C = \mathrm{Cr}_{nF} \cdot 1 \times 10^{-9} = 124.8000000000 \cdot 1 \times 10^{-9} \qquad = 0.0000001248$$

$$\mathrm{Fr} = \frac{1 \times 10^{-3}}{2 \cdot \pi \cdot \sqrt{L \cdot \mathrm{Cr}_{nF} \cdot 1 \times 10^{-9}}} = \frac{1 \times 10^{-3}}{2 \cdot 3.142 \cdot \sqrt{0.000 \cdot 124.800 \cdot 1 \times 10^{-9}}} \qquad = 50.687 \ \ (\mathrm{khz})$$

$$m = \left(\frac{R}{2}\right) \cdot \sqrt{\frac{C}{L}} = \left(\frac{3}{2}\right) \cdot \sqrt{\frac{0.000}{0.000}} \qquad = 0.060 \ \ (\text{damping factor})$$

$$\text{FreqD}_{theo} = \text{Fr} \cdot \sqrt{1 - m \cdot m} = 50.687 \cdot \sqrt{1 - 0.060 \cdot 0.060} \qquad = 50.597 \ \ (\text{kHz, damped frequency})$$

The damped frequency erorr:

$$\text{FreqD}_{meas} = 50.840$$

$$\text{Freq}_{Error} = 100 \cdot \frac{\text{FreqD}_{meas} - \text{FreqD}_{theo}}{\text{FreqD}_{theo}} = 100 \cdot \frac{50.840 - 50.597}{50.597} \qquad = 0.480 \ \ (\backslash\%)$$

The error between the tuned resonant LRC frequency and the average measured value is 0.5%, which indicates a very good estimation.

Why is it difficult to fit the LRC curve?

Reminder of the damping formulas for a series LCR circuit

Dumping factor:

$$m = \left(\frac{R}{2}\right) \cdot \sqrt{\frac{C}{L}}$$

Alpha:

$$\alpha = \omega_0 \cdot m$$

Dumping frequency:

$$F_{resd} = F_{res0} \cdot \sqrt{1 - m \cdot m}$$

$$\omega_d = \omega_0 \cdot \sqrt{1 - m \cdot m}$$

And the capacitor voltage :

$$V_0 \cdot \exp((-\alpha) \cdot t) \cdot \sin(\omega_d \cdot t - \text{phase})$$

This formula assumes that L, R, and C are constant over all voltage–current operating points, which is not accurate:

- **L:** inductance drops with increasing current due to magnetic saturation
- **R:** this is critical because R models the core losses, which strongly depend on voltage and flux density—higher current results in higher losses and thus a larger effective R
- **C:** capacitance changes slightly with voltage and tends to decrease as voltage rises, although this remains a secondary effect

Small-signal characterization

Setup

The below illustrated setup is simplified. In the actual implementation, the original DC-DC PCB was utilized with the primary and secondary MOSFETs removed. An isolated 12 V battery was inserted in series with the function generator to prevent any bootstrap capacitor loop interaction. Under these conditions, the bootstrap diode remains reverse-biased, ensuring effective isolation of the bootstrap capacitor from the measurement circuit.

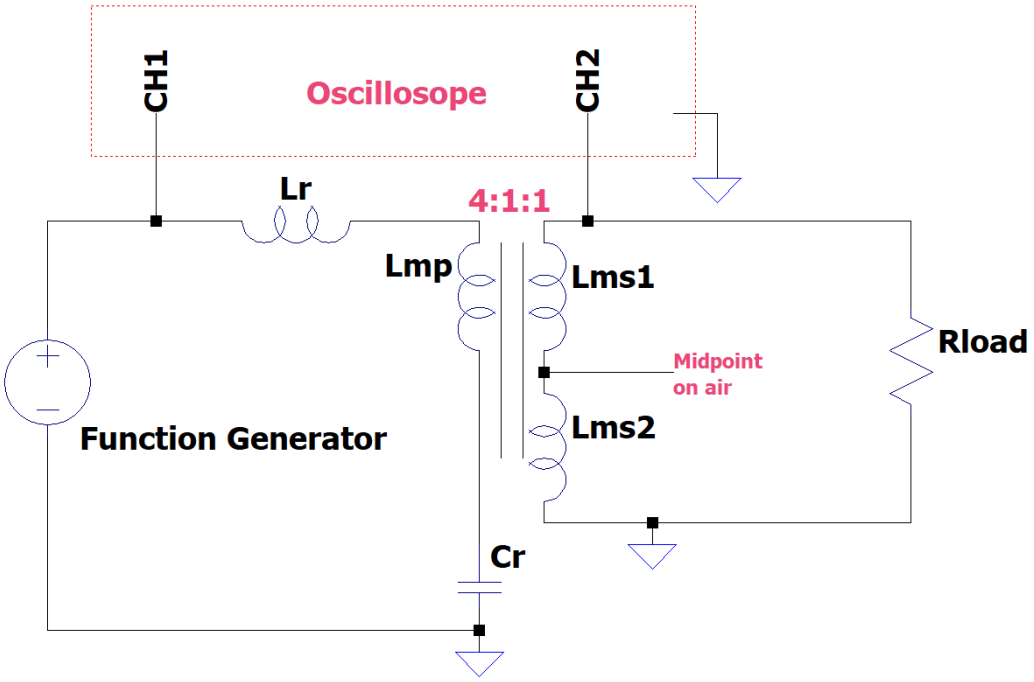


Figure 15: Small-signal characterization setup

The reference values of L, C, R...:

$$\mathrm{Lr}_{uH} = 21 \ \ (\mathrm{uH}) \qquad \mathrm{Lm}_{uH} = 83 \ \ (\mathrm{uH}) \qquad \mathrm{Cr}_{nF} = 120 \ \ (\mathrm{nF})$$

$$n = 4 \qquad R_{load} = 1963 \ \ (\Omega)$$

Calculate the Ln, Fr

$$\begin{aligned}\mathrm{Ln} &= \frac{\mathrm{Lm}_{uH}}{\mathrm{Lr}_{uH}} \\ &= \frac{83}{21} \\ &= 3.952\end{aligned}$$

$$\begin{aligned}\mathrm{Fr}_{khz} &= \frac{1 \times 10^{-3}}{2 \cdot \pi \cdot \sqrt{1 \times 10^{-6} \cdot \mathrm{Lr}_{uH} \cdot 1 \times 10^{-9} \cdot \mathrm{Cr}_{nF}}} \\ &= \frac{1 \times 10^{-3}}{2 \cdot 3.142 \cdot \sqrt{1 \times 10^{-6} \cdot 21 \cdot 1 \times 10^{-9} \cdot 120}} \\ &= 100.258 \ \ (\mathrm{kHz})\end{aligned}$$

Calculate the equivalent load in primary side

$$\mathrm{Re} = R_{load} \cdot \left(\frac{n}{2}\right)^2 = 1963 \cdot \left(\frac{4}{2}\right)^2 \qquad = 7852$$

The quality factor of the LLC

$$\mathrm{Qe} = \frac{\sqrt{1 \times 10^{3} \cdot \frac{\mathrm{Lr}_{uH}}{\mathrm{Cr}_{nF}}}}{\mathrm{Re}} = \frac{\sqrt{1 \times 10^{3} \cdot \frac{21}{120}}}{7852.00000} \qquad = 0.00168$$

Below are the measured values for the frequency, the primary peak-to-peak voltage, and the composed secondary voltage.

the gain `G` is calculated using the formula:

$$G = 0.5 \cdot n \cdot \frac{\frac{\mathrm{CH2}_{pp}}{2}}{\mathrm{CH1}_{pp}}$$

$\frac{\mathrm{CH2}_{pp}}{2}$ because the secondary voltage is composed (2 secondary winding : see the setup)

0.5 because of the half-bridge architecture

So:

$$G = \frac{\mathrm{CH2}_{pp}}{\mathrm{CH1}_{pp}}$$

Load ~2kΩ:

	Freqkhz	CH1pp	CH2pp	Load_ohm	G
0	162.10	8.45	6.52	1963	0.772
1	151.70	8.25	6.46	1963	0.783
2	139.90	7.95	6.42	1963	0.808
3	129.40	7.65	6.34	1963	0.829
4	121.30	7.40	6.22	1963	0.841
5	109.90	7.20	6.12	1963	0.850
6	101.70	6.65	6.00	1963	0.902
7	90.93	6.00	5.76	1963	0.960
8	80.99	5.02	5.52	1963	1.100
9	70.35	3.98	5.22	1963	1.312
10	61.25	2.78	4.80	1963	1.727
11	50.00	1.11	4.24	1963	3.820
12	47.00	0.42	4.00	1963	9.524
13	42.00	1.05	3.52	1963	3.352
14	37.00	2.20	3.12	1963	1.418

Reminder: The theoretical gain formula

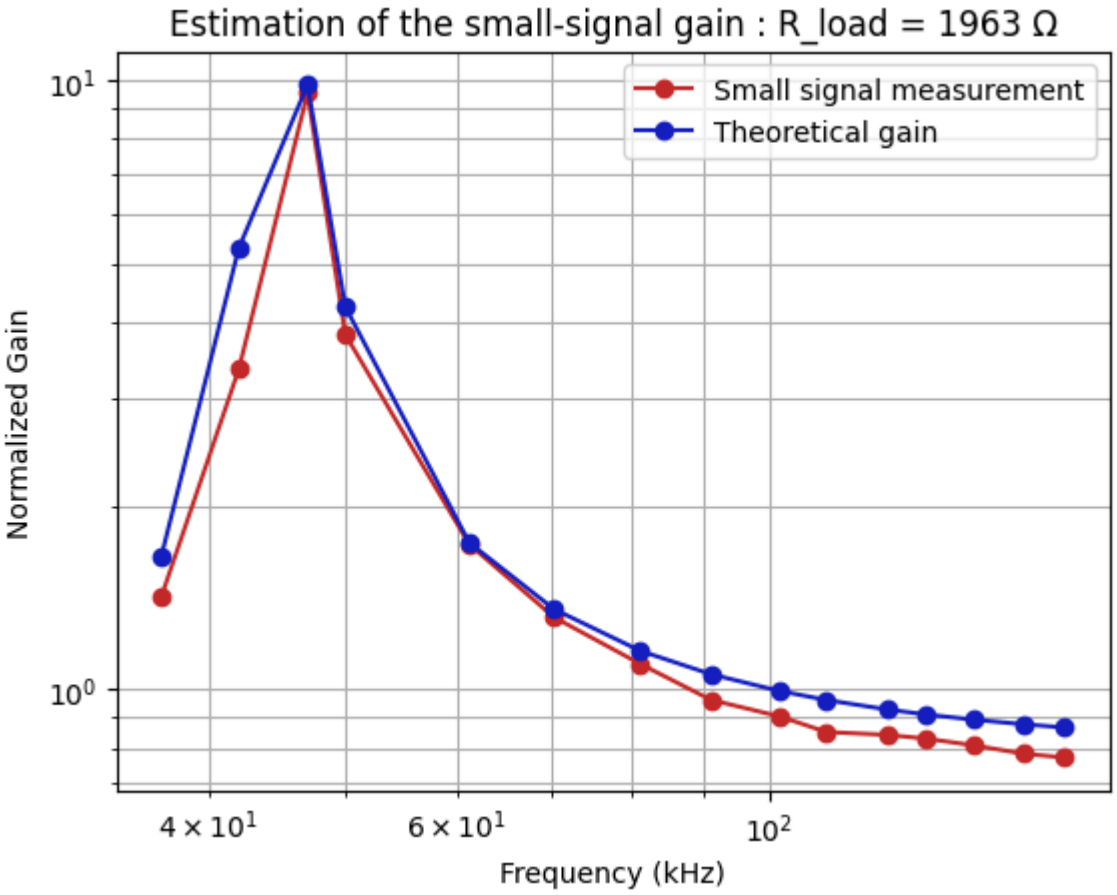
The voltage gain function (normalized) expression is:

$$M_g = \left| \frac{L_n \cdot f_n^2}{\left[(L_n + 1) \cdot f_n^2 - 1\right] + j\left[(f_n^2 - 1) \cdot f_n \cdot Q_e \cdot L_n\right]} \right|$$

With:

$$L_n = \frac{L_m}{L_r}, \quad Q_e = \frac{\sqrt{\frac{L_r}{C_r}}}{R_e}, \quad f_n = \frac{f_{sw}}{f_0}, \quad f_0 = \frac{1}{2\pi\sqrt{L_r C_r}}$$

Below is a comparison of the theoretical gain versus the measured gain.



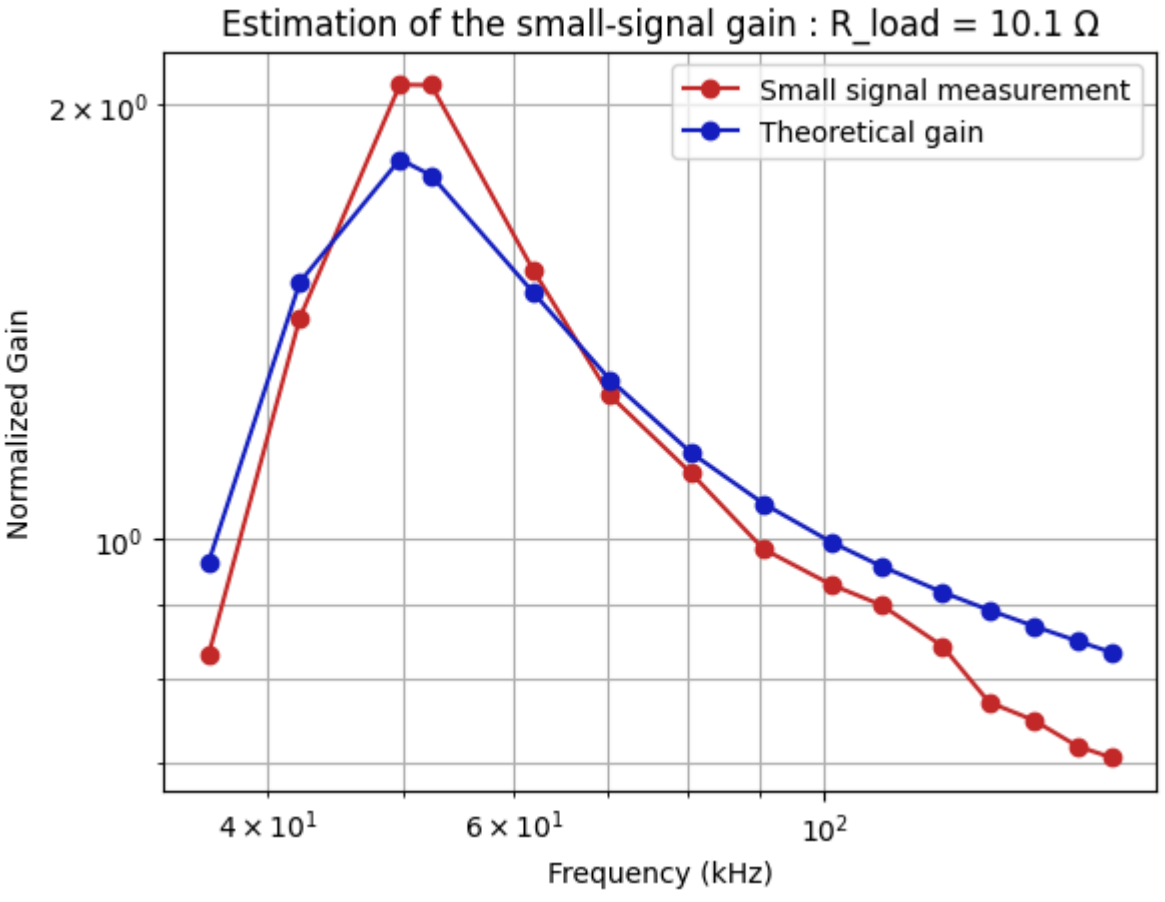
Load ~10.1Ω:

$$R_{load} = 10.1 \ \ (\Omega)$$

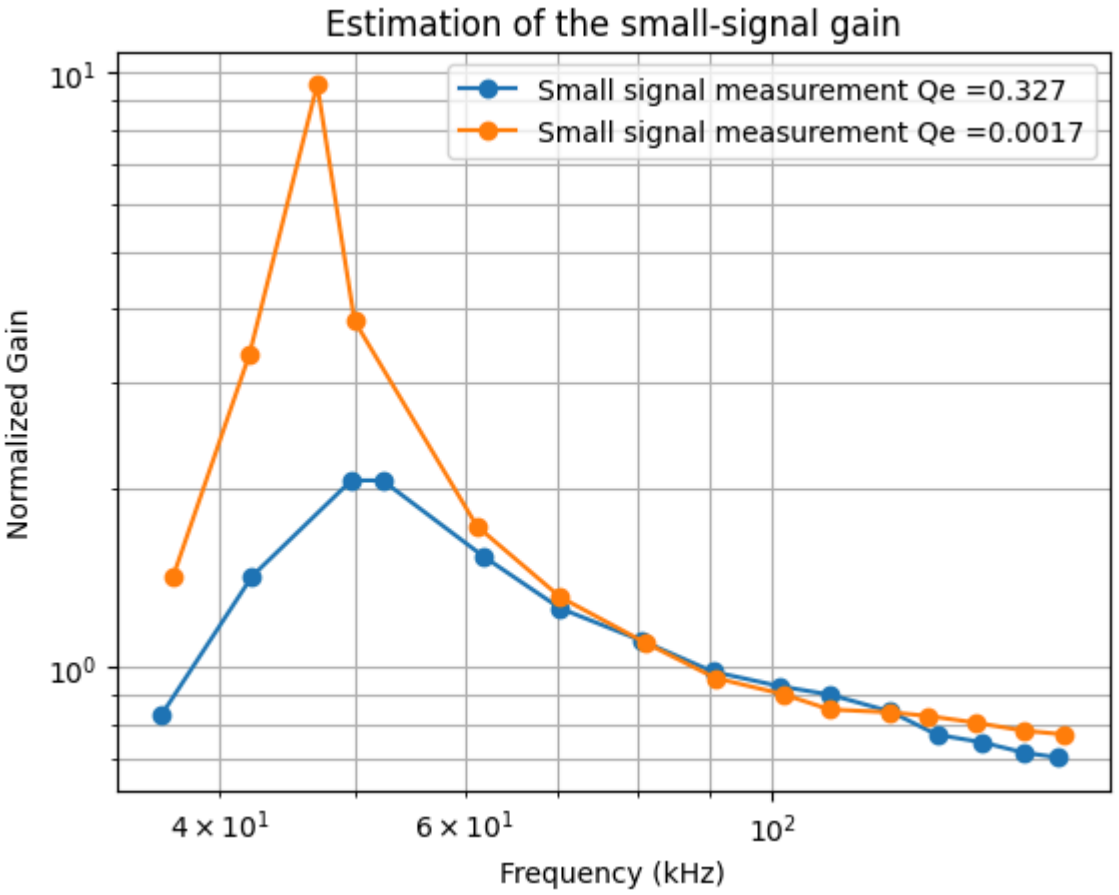
$$\mathrm{Re} = R_{load} \cdot \left(\frac{n}{2}\right)^2 = 10.10 \cdot \left(\frac{4}{2}\right)^2 \qquad = 40.40$$

$$\mathrm{Qe} = \frac{\sqrt{1 \times 10^3 \cdot \frac{\mathrm{Lr}_{uH}}{\mathrm{Cr}_{nF}}}}{\mathrm{Re}} = \frac{\sqrt{1 \times 10^3 \cdot \frac{21}{120}}}{40.400} \qquad = 0.327$$

	Freqkhz	CH1pp	CH2pp	Load_ohm	G
0	160.60	5.30	3.74	10.1	0.706
1	151.80	5.20	3.74	10.1	0.719
2	141.40	5.05	3.78	10.1	0.749
3	131.30	4.85	3.74	10.1	0.771
4	121.60	4.46	3.76	10.1	0.843
5	110.00	4.20	3.78	10.1	0.900
6	101.20	4.00	3.72	10.1	0.930
7	90.55	3.72	3.66	10.1	0.984
8	80.42	3.28	3.64	10.1	1.110
9	70.31	2.80	3.52	10.1	1.257
10	61.93	2.24	3.44	10.1	1.536
11	52.38	1.48	3.05	10.1	2.061
12	49.74	1.45	2.99	10.1	2.062
13	42.19	1.89	2.69	10.1	1.423
14	36.30	2.86	2.38	10.1	0.832



Comparing both load measurements



Operational gain measurement

Setup

In this test, we will operate the full LLC DC-DC converter using software that enables manual frequency adjustment via a potentiometer (see the setup below).

With the input voltage fixed, we will start at the maximum frequency and then reduce it progressively down to the minimum value.

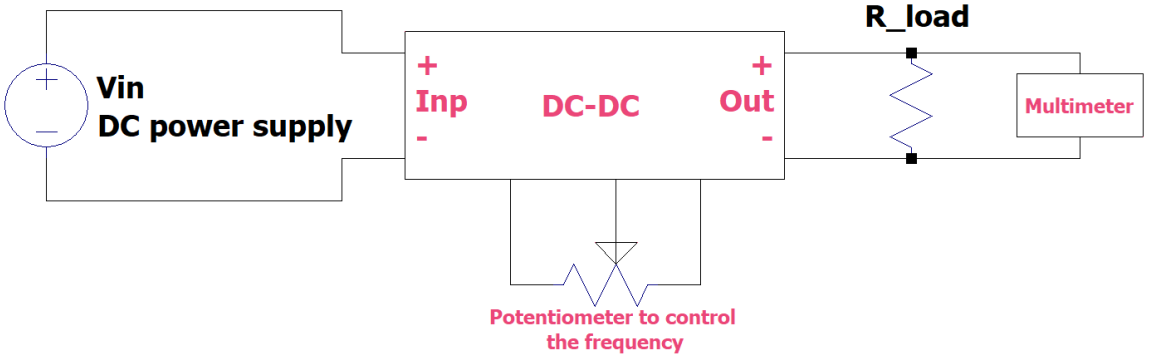


Figure 16: Operational gain measurement setup

Results

The input values

$$\mathrm{Lm}_{uH} = 83 \ \ (\mu\mathrm{H}) \qquad \mathrm{Lr}_{uH} = 21 \ \ (\mu\mathrm{H}) \qquad \mathrm{Cr}_{nF} = 120 \ \ (\mathrm{nF})$$

$$n = 4 \ \ (\mathrm{Transformator}) \qquad \mathrm{Rload} = 22 \ \ (\Omega) \qquad \mathrm{Vin} = 345 \ \ (\mathrm{V})$$

Calculation of LLC tank parameters

$$\mathrm{Ln} = \frac{\mathrm{Lm}_{uH}}{\mathrm{Lr}_{uH}} = \frac{83}{21} = 3.952$$

$$\mathrm{Re} = 8 \cdot (n)^2 \cdot \frac{\mathrm{Rload}}{(\pi)^2} = 8 \cdot (4)^2 \cdot \frac{22}{(3.142)^2} = 285.320 \ \ (\Omega)$$

$$\mathrm{Qe} = \frac{\sqrt{1 \times 10^3 \cdot \frac{\mathrm{Lr}_{uH}}{\mathrm{Cr}_{nF}}}}{\mathrm{Re}} = \frac{\sqrt{1 \times 10^3 \cdot \frac{21}{120}}}{285.320} = 0.046$$

$$\mathrm{Fr} = \frac{1}{2 \cdot \pi \cdot \sqrt{\mathrm{Lr}_{uH} \cdot 1 \times 10^{-9} \cdot \mathrm{Cr}_{nF}}} = \frac{1}{2 \cdot 3.142 \cdot \sqrt{21 \cdot 1 \times 10^{-9} \cdot 120}} = 100.258 \ \ (\mathrm{kHz})$$

We will use this formula to compute the gain. Throughout the document, the gain is indicated either as G or Mg, both referring to the same quantity.

$$\mathrm{Mg} = n \cdot \frac{\mathrm{Vout}}{\frac{\mathrm{Vin}}{2}}$$

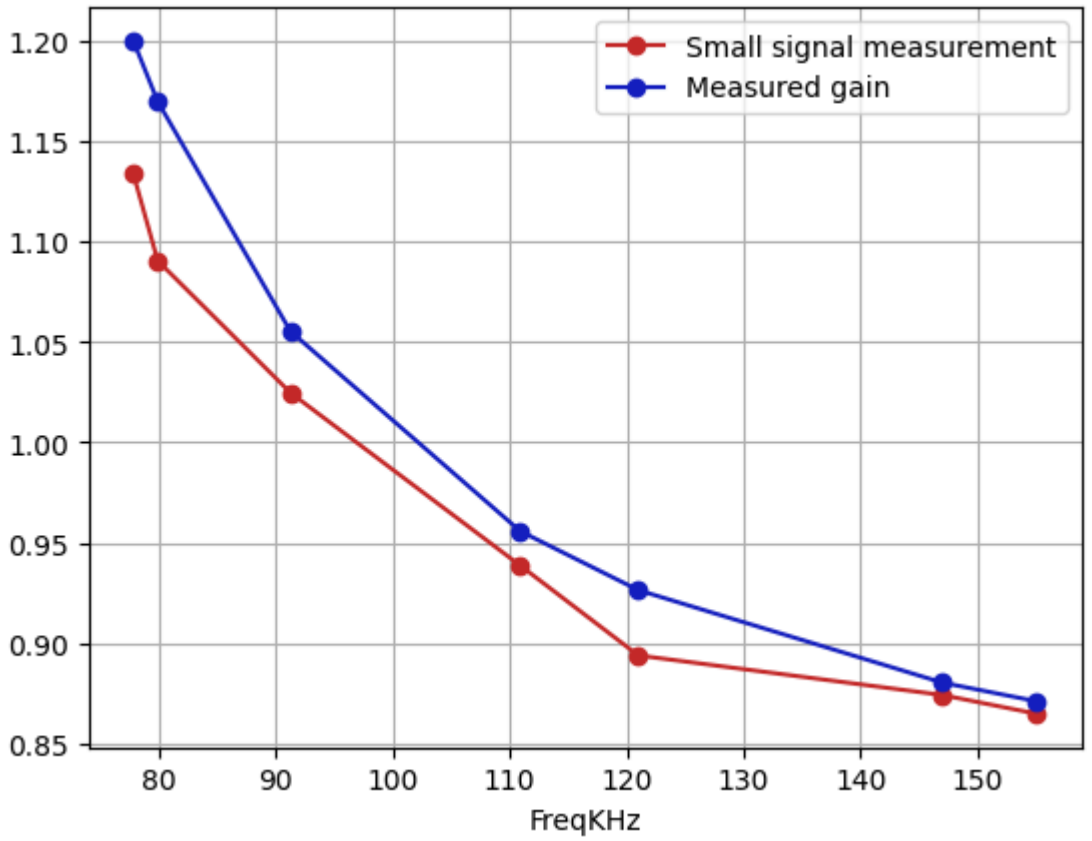
	FreqKHz	Vout	Mg
0	155.00	37.30	0.864928
1	147.00	37.70	0.874203
2	121.00	38.55	0.893913
3	110.90	40.48	0.938667
4	91.25	44.19	1.024696
5	79.91	47.02	1.090319
6	77.83	48.88	1.133449

/!\ : Starting from 80 kHz, an audible noise appears, and at 75 kHz all MOSFETs fail. This happens because ZVS is lost, causing very high losses in the primary MOSFETs and leading to a cascade of failures.

All primary/seconday mosfets are replaced.

In the future, we will limit the minimum operating frequency near the resonance point to avoid this issue (around 95 kHz).

Below is a comparison of the theoretical gain VS the measured gain.



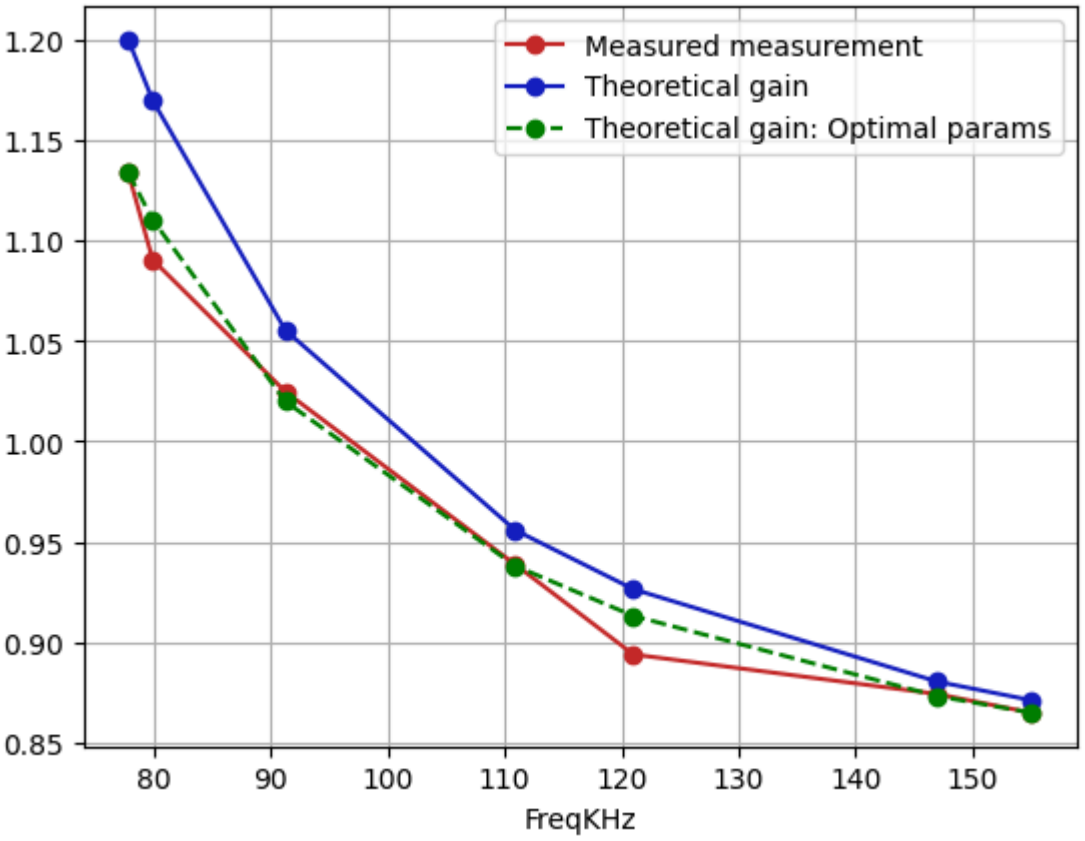
Let's try to estimate a correction to the theoretical gain to match the measurement by adjusting the quality factor **Qe**, the normalized inductance **Ln**, and a multiplicative factor **K**.

The multiplicative factor applies as follows: : Mg_opt = k * Mg

After runing the optimization python algorithm `scipy.optimize import minimize` :

	Initial values	Optimized values
Qe	0.046	0.084
Ln	3.952	4.674
k	1.000	0.975

Below the comparaison



Conclusion

As noted in the beginning, the reference values for both the transformer inductance and the resonant inductance will be based on the results of the single-pulse test.

$\mathrm{Lm}_{uH} = 83 \ \ (\mu\mathrm{H}) \qquad \mathrm{Lr}_{uH} = 21 \ \ (\mu\mathrm{H})$